1976-12

Finite element solution of a three-dimensional nonlinear reactor dynamics problem with feedback

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FINITE ELEMENT SOLUTION OF A THREE-DIMENSIONAL NONLINEAR REACTOR DYNAMICS PROBLEM WITH FEEDBACK

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NAVAL POSTGRADUATE SCHOOL

Monterey, California

THESIS

FINITE ELEMENT SOLUTION OF A THREE-DIMENSIONAL NONLINEAR REACTOR DYNAMICS PROBLEM WITH FEEDBACK

by

Eulogio Conception Bermudes

December 1976

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Approved for public release; distribution unlimited.

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER

2. GOVT ACCESSION NO.

3. RECIPIENT'S CATALOG NUMBER

4. TITLE (and Subtitle)
Finite Element Solution of a Three-Dimensional Nonlinear Reactor Dynamics Problem with Feedback

5. TYPE OF REPORT & PERIOD COVERED
Master's Thesis and Mechanical Engineer;Dec 1976

6. PERFORMING ORG. REPORT NUMBER

7. AUTHOR(s)
Eulogio Conception Bermudes

8. CONTRACT OR GRANT NUMBER(s)

9. PERFORMING ORGANIZATION NAME AND ADDRESS
Naval Postgraduate School
Monterey, California 93940

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS

11. CONTROLLING OFFICE NAME AND ADDRESS
Naval Postgraduate School
Monterey, California 93940

12. REPORT DATE
December 1976

13. NUMBER OF PAGES
160

14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)
Naval Postgraduate School
Monterey, California 93940

15. SECURITY CLASS. (of this report)
Unclassified

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

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by

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Submitted in partial fulfillment of the
requirements for the degrees of

MASTER OF SCIENCE IN MECHANICAL ENGINEERING

and

MECHANICAL ENGINEER

from the

NAVAL POSTGRADUATE SCHOOL
December 1976

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ACKNOWLEDGEMENT

The author wishes to thank Professors Dong H. Nguyen and David Salinas for their moral support and professional guidance throughout this work. Sincere appreciation is also expressed to Professor Dick Franke for his priceless assistance in the implementation of the time integration package and to Professor Giles Cantin for his invaluable counsel on finite elements. Special thanks is given to Ed Donnellan, Kris Butler, and Manus Anderson at the NPS Computer Center for their patience and cooperation in running the long computer programs required by this work. Their willingness to help and their professionalism, I am sure, are recognized and greatly appreciated by NPS students. Finally, I wish to thank my wife, Carmen, who had served outstandingly as mother and father to our children during the course of this study.

I. INTRODUCTION

The nuclear reactor with delayed neutrons and temperature-dependent feedback is a nonlinear system, whose response to uniform and local disturbances differs greatly from that of a linear reactor. The prompt temperature feedback model and one average group of delayed neutrons were incorporated in this work. Practically all neutron dynamics analysis today deals with two-dimensional geometry, implying a symmetry in neutron dynamics behavior during transient [1,2]. This symmetry assumption, however, could be unrealistic in the safety analysis of nuclear reactors. A unique feature of this work is the consideration of the three-dimensional dependence of the neutron flux. No symmetry assumptions were imposed on the problem. This work investigated neutron dynamics under uniform and local disturbances in the core. A uniform initial flux throughout the interior of the reactor was imposed. The finite element method (FEM) was employed to solve this nonlinear initial-boundary value problem. The FEM is quite effective in handling discontinuous forcing functions thereby making it particularly suited for examining the effects of localized perturbations and space-dependent feedbacks.

II. THE NUCLEAR REACTOR WITH TEMPERATURE DEPENDENT FEEDBACK AND DELAYED NEUTRONS

A. THE PHYSICAL SYSTEM

The system under consideration is a fast reactor of cylindrical geometry that is composed of two different regions. The reactor core or region I is cylinderical in shape and is fueled by U-235. Region II or the reflector region completely surrounds the core and is composed of U-238. Both regions were assumed to be homogeneous. Table I lists the physical properties and geometry for each region. A schematic of the fast reactor geomenry is shown in figure 1.

Temperature and delayed neutron effects were taken into account. Also, a one-velocity or one-group model was assumed, thereby making the velocity independent of spatial or temporal effects. The delayed neutrons were considered only in region I since region II was assumed to be a non-multiplying medium. The temperature effects were also assumed to be only in the core region.

In general form, the one-velocity neutron diffusion equation is [3]

$$\frac{\partial N(\bar{r},t)}{\partial t} = vD\nabla^2 N - \Sigma_a vN + S(\bar{r},t) \qquad (1)$$

where $\bar{r} = (x,y,z)$

D = neutron diffusion coefficient

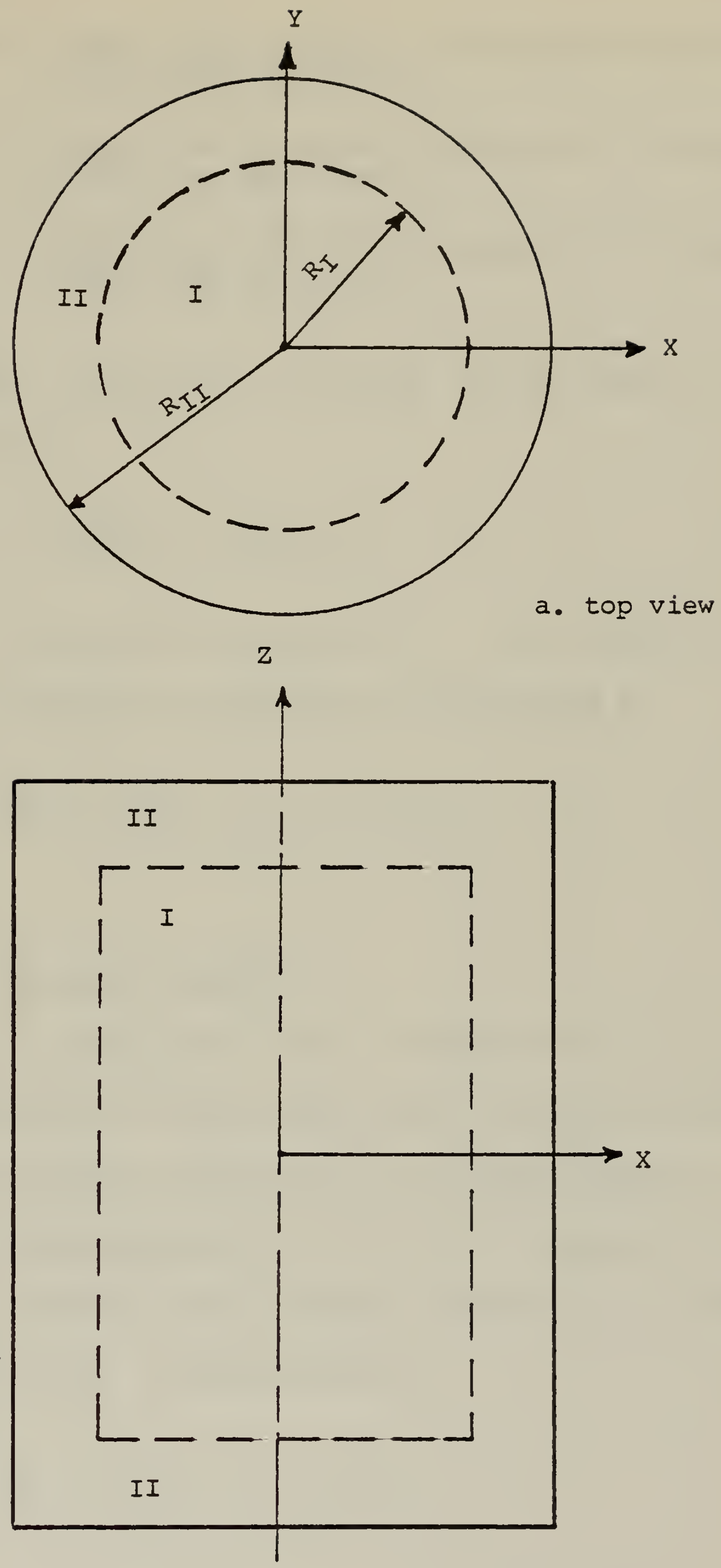
Σ_a = macroscopic absorption cross-section

$N(\bar{r},t)dV$ = number of neutrons in a volume element dV at a point $\bar{r}$ at time t

TABLE I

Physical Constants

SYMBOL	DEFINITION	VALUE
R_I	radius of Region I	60 cm
R_{II}	total reactor radius	90 cm
H_I	height of Region I	160 cm
H_{II}	total reactor height	220 cm
v	neutron speed	4.8×10^7 cm/sec
D_I	core neutron diffusion coefficient	0.913 cm
D_{II}	reflector neutron diffusion coefficient	1.200 cm
Σ_{a_I}	core neutron absorption cross section	0.01401 cm^{-1}
$\Sigma_{\alpha_{II}}$	reflector neutron absorption cross section	0.0040 cm^{-1}
ν	number of neutrons per fission	2.54
Σ_f^*	critical fission cross section	0.005736 cm^{-1}
β	delayed neutron fraction; dollar reactivity	0.00642
ε	fission energy	7.652×10^{-12} cal/fission
$\bar{h}(\frac{A}{V})$	modified convection heat transfer coefficient	0.0632 cal/(cm^2sec °C)
α	temperature coefficient	-0.004/°C
$\bar{\lambda}$	abundance-weighted mean decay constant	0.4350 sec^{-1}



a. top view

b. front view

I - core
II - reflector

Figure 1. Schematic of cylindrical reactor.

$vD\nabla^2 N dV$ = number of neutrons diffusing into dV per unit time at time t

$\Sigma_a vN \, dV$ = number of neutrons absorbed in dV per unit time at time t

$S(\bar{r},t)dV$ = number of neutrons produced in dV per unit time at time t

The neutron number density is related to the neutron flux by the expression [4]

$$\phi(\bar{r},t) = vN(\bar{r},t) \qquad (2)$$

where $\phi(\bar{r},t)$ is the flux at time t. The neutron diffusion equation in terms of the flux is depicted by

$$\frac{1}{v} \frac{\partial\phi(\bar{r},t)}{\partial t} = D\nabla^2\phi - \Sigma_a\phi + S(\bar{r},t) \qquad (3)$$

B. PROMPT AND DELAYED NEUTRONS

The source or production term in equation (3) is composed of the contributions of the prompt and delayed neutrons. The majority of the fission neutrons are prompt neutrons that appear almost instantaneously (within 10^{-7} second) on fission. Assuming a fast neutron non-leakage probability of unity, the prompt neutron source is described by

$$S_p(\bar{r},t) = (1-\beta)K_\infty(\bar{r},t)\ \phi(\bar{r},t) \qquad (4)$$

where

$K_\infty(\bar{r},t)$ = infinite medium multiplication factor

β = total fraction of delayed neutrons

The fraction of delayed neutrons is very small (note that $\beta = 0.00642$ from Table I). However, they have a very significant effect on the reactivity because their mean lifetimes are long. Without delayed neutrons, reactor control would not be possible. These delayed neutrons are born in the decay by neutron emission of nuclei produced following the β-decay of certain fission fragments. For example, the β-decay of the fission fragment Br^{87} leads to Kr^{86} plus a neutron. Nuclei such as Br^{87} whose production in fission eventually leads to the emission of a delayed neutron are called delayed neutron precursors [4].

There are six main groups of delayed neutrons. Each group is classified according to its decay constant. The delayed neutron source term is portrayed by

$$S_d(\bar{r},t) = \sum_{i=1}^{6} C_i(\bar{r},t)\ \lambda_i \qquad (5)$$

where

λ_i = decay constant of the i^{th} group

$C_i(\bar{r},t)$ = density of the i^{th} precursor

Assuming that the fission fragments do not migrate appreciable distances and assuming a non-circulating fuel reactor [3], the precursor density is delineated by

$$\frac{\partial C_i(\bar{r},t)}{\partial t} = \beta_i K_\infty \Sigma_a \phi - \lambda_i C_i \qquad (6)$$

where β_i is the fraction of delayed neutrons of the i^{th}

group. The solution to the precursor equation is in terms of a time integral expressed by

$$C_i(\bar{r},t) = \beta_i\Sigma_a \int_0^t e^{-\lambda_i(t-t')}K_\infty(\bar{r},t)\phi(\bar{r},t)dt' \qquad (7)$$

Inserting equation (7) in equation (5) yields the delayed neutron production term as

$$S_d(\bar{r},t) = \sum_{i=1}^{6} \beta_i\lambda_i\Sigma_a \int_0^t e^{-\lambda_i(t-t')}K_\infty(\bar{r},t)\phi(\bar{r},t)dt' \qquad (8)$$

For convenience the six main groups of delayed neutrons were considered as one group. This was accomplished by using the abundance-weighted mean decay constant defined by

$$\bar{\lambda} = \frac{1}{\beta}\sum_{i=1}^{6} \beta_i\lambda_i \qquad (9)$$

This is a reasonable approximation since many of the important phenomena of nuclear reactor dynamics can be characterized satisfactorily by combining all the emitters or precursors into one, two, or, at most, three effective groups [3]. Replacing λ_i with the abundance-weighted mean decay constant showed the delayed neutron source term to be

$$S_d(\bar{r},t) = \beta\bar{\lambda}\Sigma_a \int_0^t e^{-\bar{\lambda}(t-t')}K_\infty(\bar{r},t)\phi(\bar{r},t)dt' \qquad (10)$$

C. DOPPLER EFFECT AND TEMPERATURE FEEDBACK MODEL

The Doppler effect in fast reactors is due to the temperature broadening of many closely spaced high-energy resonances in both the fission and parasitic-absorption cross-sections.

These resonances basically mean that as the temperature increases, the number of neutrons that are absorbed increases and, similarly, the number of fission events increases. These nonproductive and productive processes compete in a complicated manner, and the net effect may be either an increase or decrease in reactivity [3]. For most fast reactors, the effect is negative, and it will be so assumed in this work.

The reactivity change with respect to the fuel temperature change is modeled by [2]

$$\frac{dK_\infty}{dT} = aT^{-3/2} + bT^{-1} + cT^{(m-1)} \qquad (11)$$

where a, b, and c are parameters determined from experimental or neutronic calculations, m is an integer, and T is the current fuel temperature. For most fast reactor cores (ceramic-fueled cores), TdK_∞/dT is very close to being constant over a wide range of temperatures. Therefore, it is assumed that a and c are zero and the Doppler coefficient is expressed by

$$b = T\frac{dK_\infty}{dT} \qquad (12)$$

The following initial conditions were used:

$$K_\infty(\bar{r},0^+) = K_\infty^\circ \qquad (12a)$$

$$T(\bar{r},0^+) = T_o \qquad (12b)$$

The reactivity model is then expressed by

$$K_\infty(\bar{r},t) = K_\infty^\circ + b \; \ell n \; (\frac{T}{T_o}) \qquad (13)$$

where

K_∞° = multiplication factor at steady-state

ν = number of neutrons produced per fission

T_o = original fuel temperatures

b = Doppler constant

The definition of K_∞° is

$$K_\infty^\circ = \frac{\nu\Sigma_f^{\ *}}{\Sigma_a^{\ c}} \qquad (14)$$

where

$\Sigma_f^{\ *}$ = critical fission cross-section

$\Sigma_a^{\ c}$ = macroscopic absorption cross-section of the core

The rise in fuel temperature is depicted by

$$\theta(\bar{r},t) = T(\bar{r},t) - T_o(\bar{r}) \qquad (15)$$

This is also described by the integral [5]

$$\theta(\bar{r},t) = \int_o^t f(t-t') \; \psi(\bar{r},t)dt' \qquad (16)$$

where $f(t-t')$ is the feedback kernel and is dependent upon the type of temperature feedback model used. $\psi(\bar{r},t)$ is the

rise in neutron flux above the steady state and is expressed by

$$\psi(\bar{r},t) = \phi(\bar{r},t) - \phi_o(r) \tag{17}$$

where $\phi_o(r)$ is the neutron flux at steady state.

There are three types of temperature feedback models that can be considered here. The first is known as "Newton's feedback" which determines the reactor temperature by Newton's law of cooling. The second temperature feedback model is called the adiabatic feedback model. This represents the temperature for the loss of coolant case. The third model is the prompt temperature feedback model in which the fuel temperature follows the behavior of the neutron flux without delay [5,6]. The prompt feedback model was employed in this analysis. The feedback kernel for this temperature feedback model is

$$f(t-t') = \frac{K}{\gamma}\,\delta(t-t') \tag{18}$$

where K is an energy production operator with units of °C per unit flux and γ , a dimensionless quantity, is related to the mean time for heat transfer to coolant.

Inserting equation (18) in equation (16) and performing the integration produced a rise in temperature of

$$\theta(\bar{r},t) = \frac{K}{\gamma}\,\psi(\bar{r},t) \tag{19}$$

From equation (19) the temperature of the fuel is

$$T(\bar{r},t) = \frac{K}{\gamma}\,\psi(\bar{r},t) + T_o(\bar{r}) \qquad (20)$$

The ratio of the current temperature to the steady-state temperature is therefore

$$\frac{T}{T_o} = \frac{K}{\gamma T_o}\,\psi + 1 \qquad (21)$$

In this work T_o was considered to be constant throughout the core. Incorporating equation (21) into equation (13) gave the reactivity model of

$$K_\infty(\bar{r},t) = K_\infty^\circ + b\;\ell n\left[\frac{K}{\gamma T_o}\;\psi(\bar{r},t) + 1\right] \qquad (22)$$

D. FIELD EQUATIONS

Before establishing the field equations it is desirable to express equation (3) in terms of the rise in flux. Using equation (17) in equation (3) and grouping terms yielded the following diffusion equation:

$$\frac{1}{v}\,\frac{\partial\psi}{\partial t} = \left[D\nabla^2\psi - \Sigma_a\psi + (1-\beta)K_\infty\Sigma_a\psi + \beta\bar{\lambda}\Sigma_a\int_0^t e^{-\bar{\lambda}(t-t')}K_\infty\psi\,dt'\right]$$

$$+ \left[D\nabla^2\phi_o - \Sigma_a\phi_o + (1-\beta)K_\infty\Sigma_a\phi_o + \beta\bar{\lambda}\Sigma_a\int_0^t e^{-\bar{\lambda}(t-t')}K_\infty\phi_o\,dt'\right]$$

$$(23)$$

The second bracketed term in equation (23) is identically equal to zero since it is the steady state portion of the

diffusion equation. The rise in neutron flux above its steady state value is therefore expressed, for the core, by

$$\frac{1}{v}\frac{\partial\psi}{\partial t} = D\nabla^2\psi - \Sigma_a\psi + (1-\beta)\, K_\infty\Sigma_a\psi$$

$$+ \beta\bar{\lambda}\Sigma_a \int_0^t e^{-\bar{\lambda}(t-t')}\, K_\infty\psi\, dt' \qquad (24)$$

and, for the reflector region, by

$$\frac{1}{v}\frac{\partial\psi}{\partial t} = D\nabla^2\psi - \Sigma_a\psi \qquad (25)$$

Inserting the reactivity model into equation (24) yielded

$$\frac{1}{v}\frac{\partial\psi}{\partial t} = D\nabla^2\psi - \Sigma_a\psi + (1-\beta)\,\Sigma_a K_\infty^\circ\psi$$

$$+(1-\beta)b\Sigma_a\ \{\ell n[\frac{K\psi}{\gamma T_o} + 1]\}\ \psi + \beta\bar{\lambda}K_\infty^\circ \int_0^t e^{-\bar{\lambda}(t-t')}\psi dt'$$

$$+ \beta\bar{\lambda}\Sigma_a b\ \{\int_0^t e^{-\bar{\lambda}(t-t')}[\ell n(\frac{K\psi}{\gamma T_o} + 1)]\ \psi dt'\} \qquad (26)$$

For the reflector, the last four terms of equation (26) are zero. The effects of the temperature on the delayed neutrons were neglected in this work. The field equations can now be expressed, for the core, as

$$\frac{\partial\psi}{\partial t} - vD\nabla^2\psi + [v\Sigma_a - v(1-\beta)\ \nu\Sigma_f]\psi$$

$$+ [-(1-\beta)v\Sigma_a b]\ [\ell n(\frac{K\psi}{\gamma T_o} + 1)]\psi$$

$$+ [-\beta\bar{\lambda}v\Sigma_f\nu]\ [\int_0^t e^{-\bar{\lambda}(t-t')}\psi dt'] = 0 \qquad (27)$$

and, for the reflector, as

$$\frac{\partial\psi}{\partial t} - vD\nabla^2\psi + v\Sigma_a\psi = 0 \qquad (28)$$

The non-linear terms of the core field equation will be linearized accordingly. In more compact form, equations (27) and (28) became

$$\frac{\partial\psi}{\partial t} - c1\nabla^2\psi + c2\psi + c4[\ell n(\frac{K\psi}{\gamma T_o} +1)]\psi+c5[\int_o^t e^{-\bar{\lambda}(t-t')}\psi dt']=0 \qquad (29)$$

and

$$\frac{\partial\psi}{\partial t} - c1\nabla^2\psi + c2\psi = 0 \qquad (30)$$

where the meanings of the coefficients c1, c2, etc., are obvious for the core and reflector.

Equations (29) and (30) were subjected to the following conditions:

boundary condition:

$$\psi_R(\bar{r}_B,t) = 0 \qquad (31)$$

where $\bar{r}_B$ are coordinates of points on the outer surface of the reflector and the subscript R refers to the reflector

continuity of flux:

$$\psi_R(\bar{r},t) = \psi_c(\bar{r}_I,t) \qquad (32)$$

where $\bar{r}_I$ are coordinates of points on the core-reflector interface and the subscript c refers to the core.

III. FINITE ELEMENT

A. INTRODUCTION

The application of the finite elements of nonlinear continua has been mostly in the field of solid mechanics. Prior work [1] using the finite elements of nonlinear continua on a nonlinear reactor dynamics problem has been successful. The FEM in this work was utilized to reduce a nonlinear partial differential equation of the nuclear reactor to a system of nonlinear ordinary differential equations in time. The time integration was accomplished by using a computer program for the numerical solution of stiff differential equations developed by Franke [7]. In order to minimize computer storage requirements, an optimum compacting scheme (OCS), described by Ref. 8, was adopted.

The finite element models of operator equations are generally classified into three categories: a) the variational finite element models such as the Ritz method, b) the weighted residuals method such as the method of Galerkin, and c) the direct finite element models which are not based on functional minimization. From experience in structural mechanics, the most effective method for generating acceptable finite element models of nonlinear equations is the Galerkin method [1]. This work adopted the method of Galerkin in a finite element approximation over the spatial domain of the field equations.

B. THE METHOD OF GALERKIN

The Galerkin method is a special case of the method of weighted residuals. It involves a rational choice of weighting function that is consistent with the type of finite element approximation considered. Indeed, the weighting functions chosen are the basis or shape functions employed in the finite element approximation. There are two favorable characteristics of the Galerkin method which makes it attractive. The first attribute is its amenability to integration by parts. This supplied the freedom of using a lower order finite element than might be otherwise possible. The second favorable characteristic of the Galerkin method is that the symmetric operators in the field equations transform into symmetric matrix operators. Both these attributes are attractive for computational purposes.

Consider the initial-boundary-value problem

$$\frac{\partial\psi}{\partial t} = \mathcal{L}\psi - f(\bar{r},t) \qquad (33)$$

where $\mathcal{L}$ contains the nonlinear operators. According to the spatial finite element discretization, the solution of equation (33) is in the form of an union of an $\bar{N}$-term approximation given by

$$\psi(\bar{r},t) \approx \tilde{\psi}(\bar{r},t) = \bigcup_{J=1}^{\bar{N}} \psi_J(t)\, G_J(\bar{r}) \;, \quad J=1,2,\ldots,\bar{N} \qquad (34)$$

where $\bar{N}$ is the number of system degrees of freedom (i.e., number of coordinates), and $G_J(\bar{r})$ are the system or global basis functions which span the space of the approximate

solution $\tilde{\psi}(\bar{r},t)$ [1]. The Einstein summation is used. The global basic functions are "pyramid functions", each of which has a prescribed functional description over a subdomain of the system and is zero elsewhere [9]. The unknown coordinate functions $\psi_J(t)$ are the time-dependent magnitudes of the approximated flux $\tilde{\psi}(\bar{r},t)$ and/or its derivatives at discrete nodal points [10].

The residual function, $R(\bar{r},t)$, is defined such that it is identical to zero when $\tilde{\psi}(\bar{r},t)$ is equal to the exact solution. The residual function is expressed by

$$R(\bar{r},t) = \frac{\partial\tilde{\psi}}{\partial t} - \mathcal{L}\tilde{\psi} - f \tag{35}$$

The Galerkin orthogonality condition (using the basis functions as weight functions), when applied to the residual function, requires that

$$\int_{Vol} G_I R(\bar{r},t)dVol = 0\ , \qquad I=1,2,\ldots,\bar{N} \tag{36}$$

From the field equations, the residual function for the core is

$$R(\bar{r},t) = \frac{\partial\tilde{\psi}}{\partial t} - c1\nabla^2\tilde{\psi} + c2\tilde{\psi} + c4[\ell n(\frac{K\tilde{\psi}}{\gamma T_o} + 1)]\tilde{\psi}$$

$$+ c5[\int_0^t e^{-\bar{\lambda}(t-t')}\,\tilde{\psi}dt'] \tag{37}$$

and for the reflector

$$R(\bar{r},t) = \frac{\partial\tilde{\psi}}{\partial t} - c1\nabla^2\tilde{\psi} + c2\tilde{\psi} \tag{38}$$

Using equation (34) and applying Galerkin's orthogonality condition produced a core equation of

$$\int_{Vol} G_I\{G_J\dot{\psi}_J(t)-\nabla^2 G_J(c1\cdot\psi)_J + G_J(c2\cdot\psi)_J$$

$$+[\ell n(\frac{K}{\gamma T_o} G_K\psi_K + 1)]\ G_J(c4\cdot\psi)_J$$

$$+[\int_0^t e^{-\bar{\lambda}(t-t')}(c5\cdot\psi)_J\ dt']G_J\} = 0 \qquad (39)$$

where $I = 1,2,\ldots,\bar{N}$

$J = 1,2,\ldots,\bar{N}$

$K = 1,2,\ldots,\bar{N}$

The reflector has a similar equation without the nonlinearity and is expressed by

$$\int_{Vol} G_I\{G_J\dot{\psi}_J(t) - \nabla^2 G_J(c1\cdot\psi)_J + G_J(c2\cdot\psi)_J\}\ dVol = 0 \qquad (40)$$

C. THE ELEMENT

A three-dimensional quadratic isoparametric element was employed in this work. The parent element is a triangular prism or solid wedge with straight sides. The element shape functions are expressed in terms of area coordinates in the plane of the triangle and by an isoparametric coordinate

along the prism axis. Figure 2 shows the parent element. This element was chosen because of the ease with which it fits the cylindrical structure when it is transformed into a curved element. This type of element has been used before as filler elements [11].

The area coordinates are defined by area ratios. Consider the triangle shown in figure 3. An arbitrary point P within the triangle defines three subareas designated by A_1, A_2, and A_3. The ratio of each of the subareas to the total area is known as an area coordinate. In equation form, the area coordinates are

$$L_1 = A_1/A \tag{41a}$$

$$L_2 = A_2/A \tag{41b}$$

$$L_3 = A_3/A \tag{41c}$$

where A is total area of the triangle. L_1, L_2, and L_3 are the natural coordinates for a triangle. The requirement that the sum of the subareas be equal to the total area is obviously satisfied by the identity

$$L_1 + L_2 + L_3 = 1 \tag{42}$$

In the plane of the triangle, only two of the area coordinates are independent.

The isoparametric coordinates are best visualized by considering the rectangular prism shown in figure 4. Isoparametric coordinates are normalized coordinates such that their values on the faces of the rectangle are $\pm$ 1. The $\xi\eta\zeta$ axes are in general not orthogonal. They are orthogonal

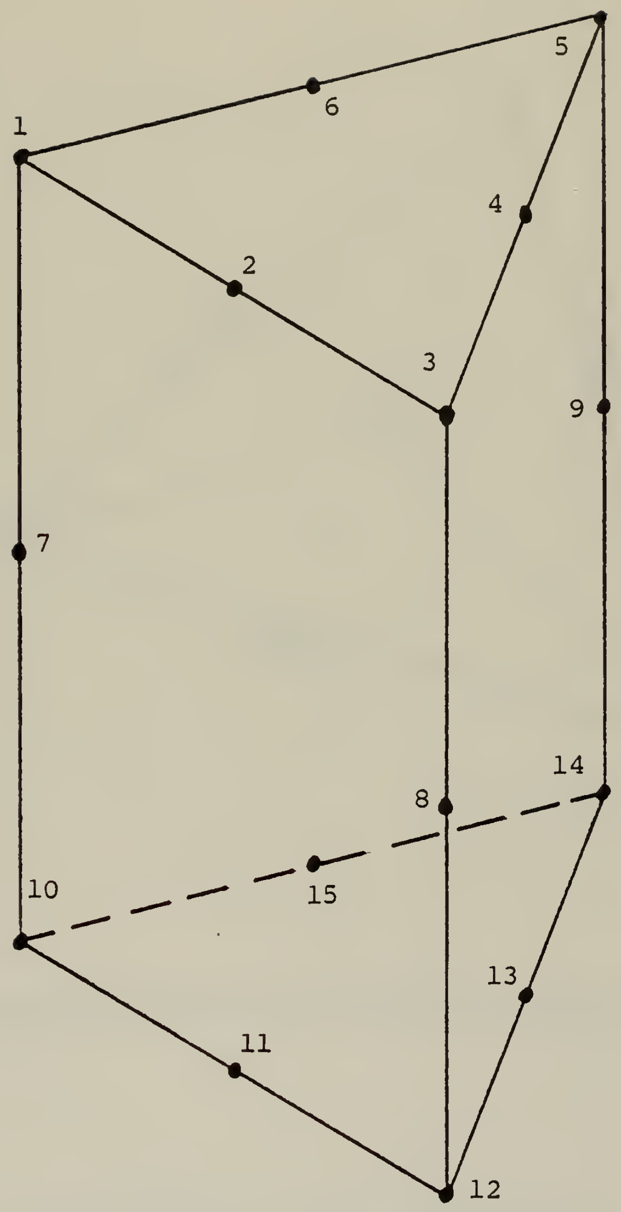


Figure 2. Quadratic triangular prism parent element

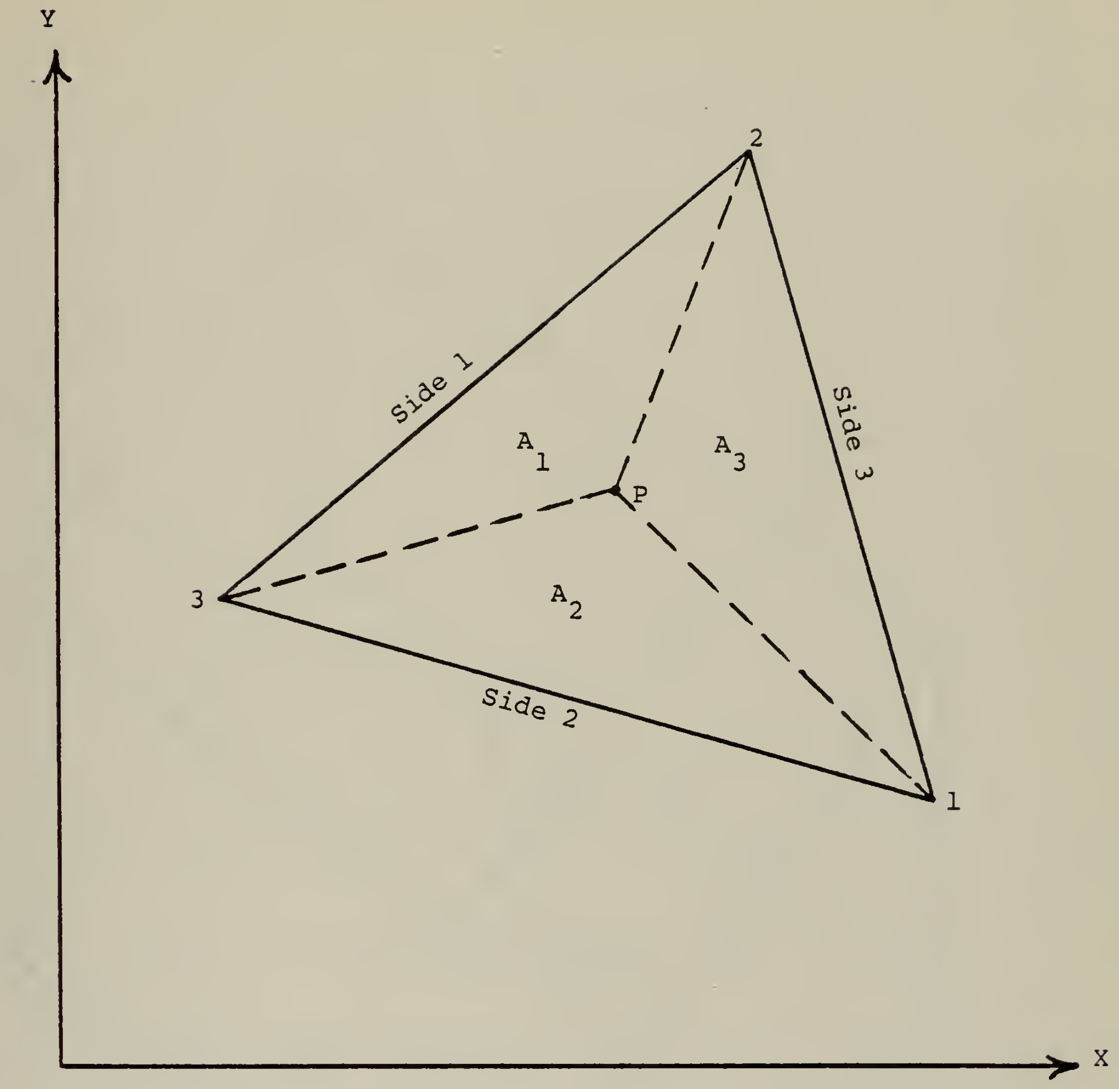


Figure 3. Definition of area coordinates

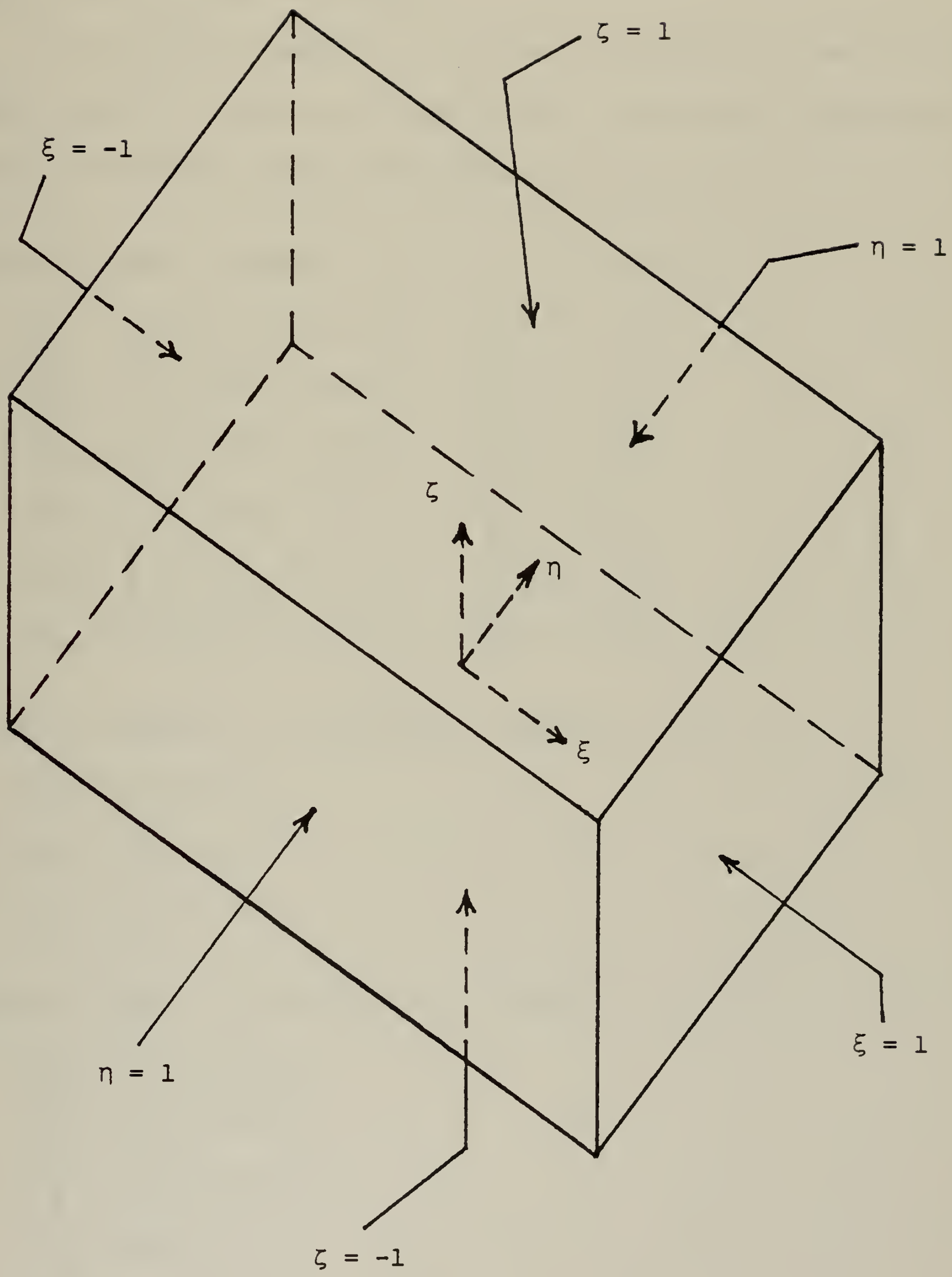


Figure 4. Isoparametric coordinates

only in the special case of a rectangular prism element [12]. The element basis functions use the ζ coordinate in the prism axis and the L_1, L_2, and L_3 coordinates in the plane of the triangle.

The parent element, as shown in figure 2, has 15 local nodal points around its periphery. As such, there are 15 basis functions which are given below [11]:

Corner nodes: (nodes 1, 3, 5, 10, 12, 14)

$$N_1 = \frac{1}{2} L_1(2L_1-1)(1+\zeta) - \frac{1}{2} L_1(1-\zeta^2)$$
$$N_3 = \frac{1}{2} L_2(2L_2-1)(1+\zeta) - \frac{1}{2} L_2(1-\zeta^2)$$
$$N_5 = \frac{1}{2} L_3(2L_3-1)(1+\zeta) - \frac{1}{2} L_3(1-\zeta^2)$$
$$N_{10} = \frac{1}{2} L_1(2L_1-1)(1-\zeta) - \frac{1}{2} L_1(1-\zeta^2)$$
$$N_{12} = \frac{1}{2} L_2(2L_2-1)(1-\zeta) - \frac{1}{2} L_2(1-\zeta^2)$$
$$N_{14} = \frac{1}{2} L_3(2L_3-1)(1-\zeta) - \frac{1}{2} L_3(1-\zeta^2)$$

Midside nodes of rectangles: (nodes 7, 8, 9)

$$N_7 = L_1(1-\zeta^2)$$
$$N_8 = L_2(1-\zeta^2)$$
$$N_9 = L_3(1-\zeta^2)$$

Midside nodes of triangles: (nodes 2, 4, 6, 11, 13, 15)

$$N_2 = 2L_1L_2(1+\zeta)$$
$$N_4 = 2L_2L_3(1+\zeta)$$
$$N_6 = 2L_3L_1(1+\zeta)$$
$$N_{11} = 2L_1L_2(1-\zeta)$$
$$N_{13} = 2L_2L_3(1-\zeta)$$
$$N_{15} = 2L_3L_1(1-\zeta)$$

The coordinates of each local node, in terms of L_1, L_2, L_3, and ζ are listed in table II. These element basis functions < N > define the geometry of the element. Note that they satisfy the relationship

$$N_i = \begin{cases} 1 \text{ , at node } i \\ \\ 0 \text{ , at other nodes} \end{cases} \qquad (43)$$

On the element level, the variation of the unknown function ψ is approximated by

$$\tilde{\psi}^e = < N '> \{\psi\}^e \qquad (44)$$

where < N'> = row vector of element shape functions

$\{ \psi \}^e$ = column vector of time-dependent nodal values of $\tilde{\psi}^e$

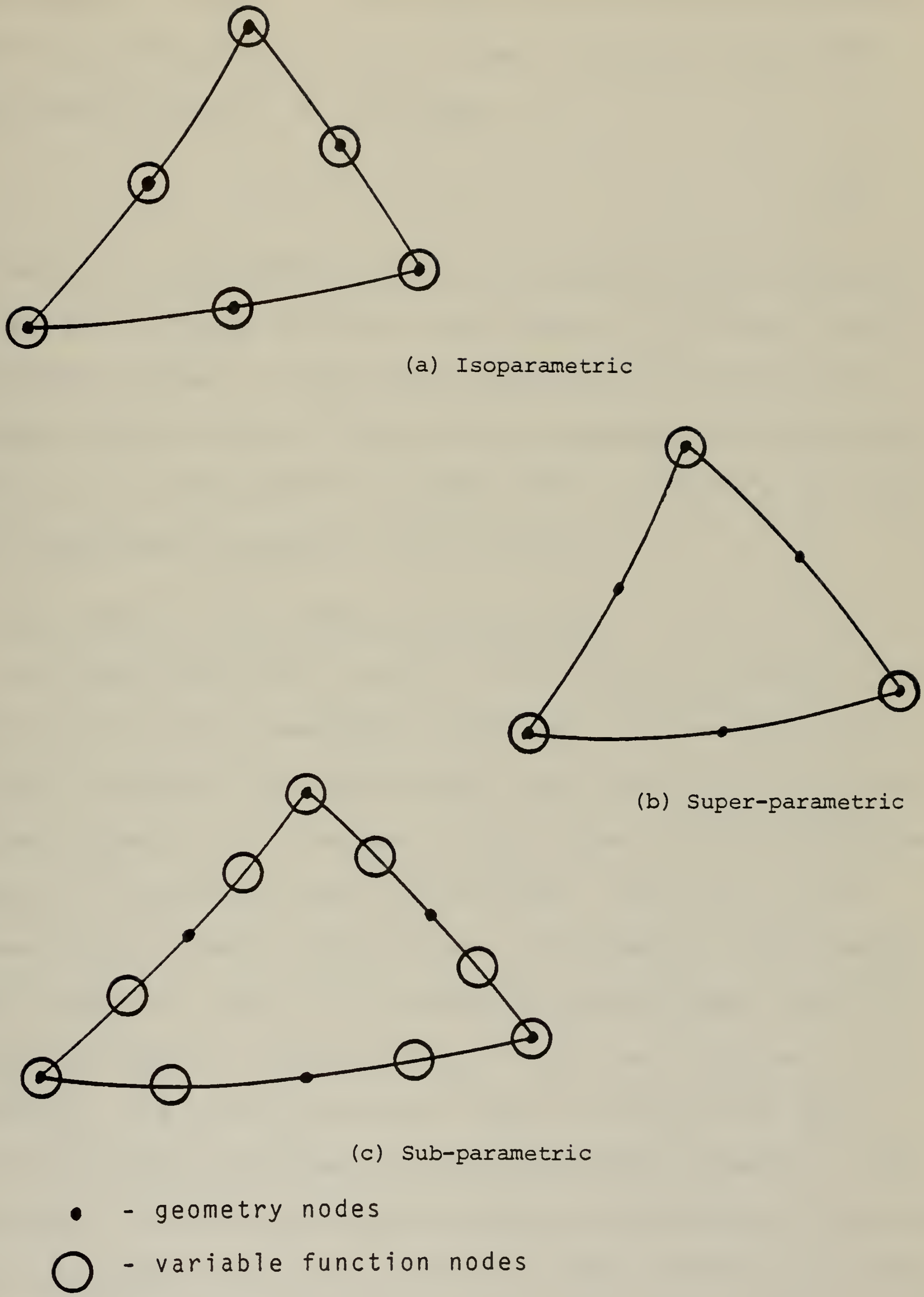
To satisfy continuity requirements, the shape functions < N'> have to be such that the continuity of the unknown function ψ is preserved in the parent coordinates [11].

The shape functions < N > which characterize the element geometry and the shape functions < N'> which describe the unknown function do not necessarily have to be the same. There is no requirement that the nodal values be associated with the same nodes which were used to define the element geometry, though in practice it is often the case. Consider for example, the illustrations in figure 5, If the nodes defining the element geometry and the nodes defining the

TABLE II

Coordinates of Local Nodal Points

Local Node	L_1	L_2	L_3	5
1	1	0	0	1
2	½	½	0	1
3	0	1	0	1
4	0	½	½	1
5	0	0	1	1
6	½	0	½	1
7	1	0	0	0
8	0	1	0	0
9	0	0	1	0
10	1	0	0	-1
11	½	½	0	-1
12	0	1	0	-1
13	0	½	½	-1
14	0	0	1	-1
15	½	0	½	-1



(a) Isoparametric

(b) Super-parametric

(c) Sub-parametric

Figure 5. Element classification

unknown function are identical, the element is known as an isoparametric element. This means that the shape functions describing the geometry and the shape functions describing the unknown function ψ are equal, or

$$< N' > = < N > \tag{45}$$

If there are more nodes defining the geometry than nodes defining the variable function, the element is called a superparametric element. Using more nodal points to define the unknown function than to describe the geometry of the element results in a subparametric element [11]. This work utilized the isoparametric element classification.

D. DIVISION OF THE SYSTEM INTO ELEMENTS

In three-dimensional space, the division of the system into discrete elements is difficult to visualize. It is virtually impossible to show every nodal point of the system in one schematic. To present a clear view of the discretized domain, a "layer" approach was adopted. The first finite element grid or mesh employed here consisted of 128 elements. Under this grid (mesh I), the reactor was divided into four layers as shown in figure 6. Each layer was composed of 32 elements. The first and fourth layers were each 30 cm in height and each contained entirely reflector elements. The second and third layers were each 80 cm in height and together they encompassed the entire core plus the remaining reflector elements. Each layer, in turn, was partitioned into three horizontal (xy) planes. The top plane included all the global

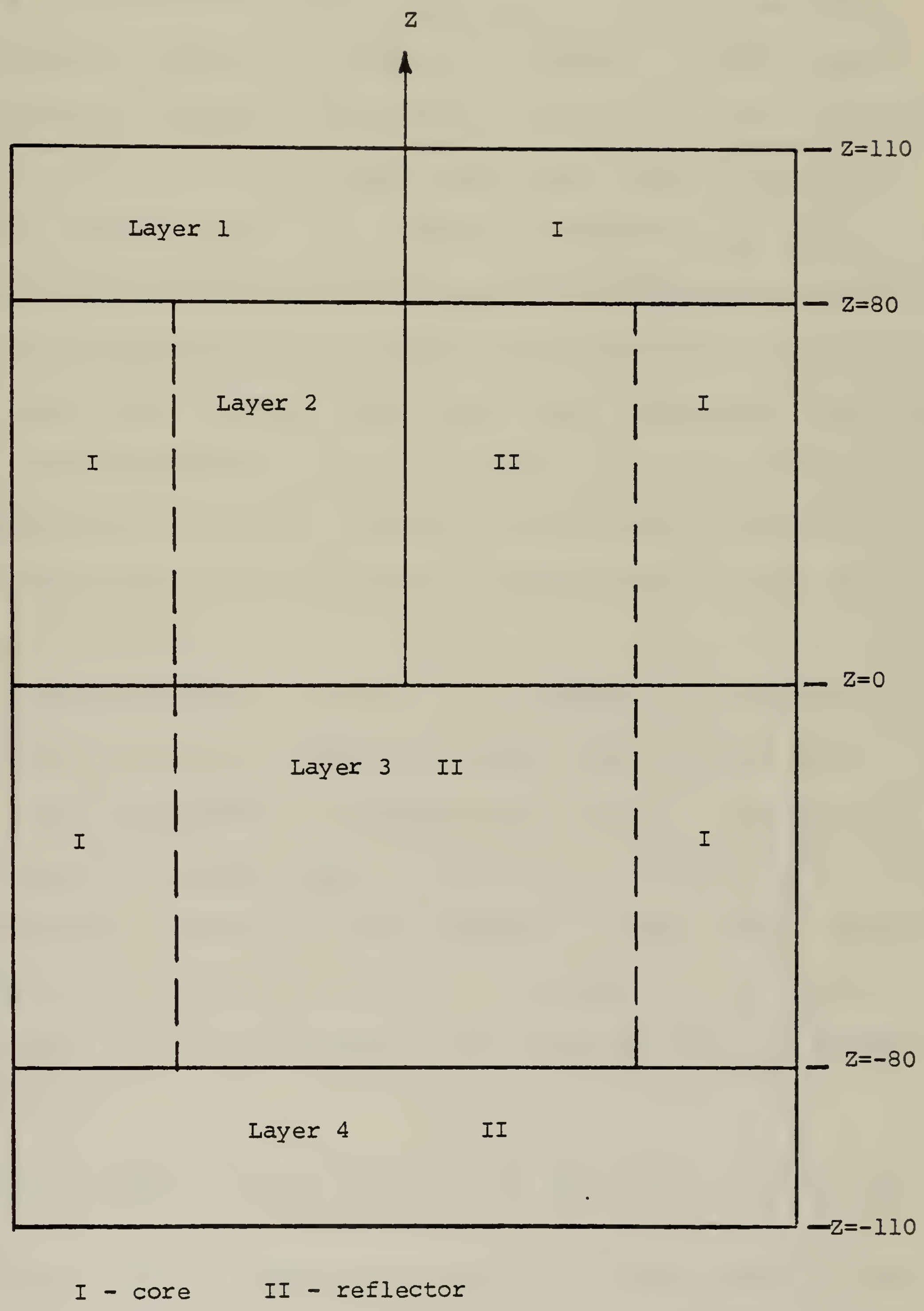


Figure 6. Layers of mesh I

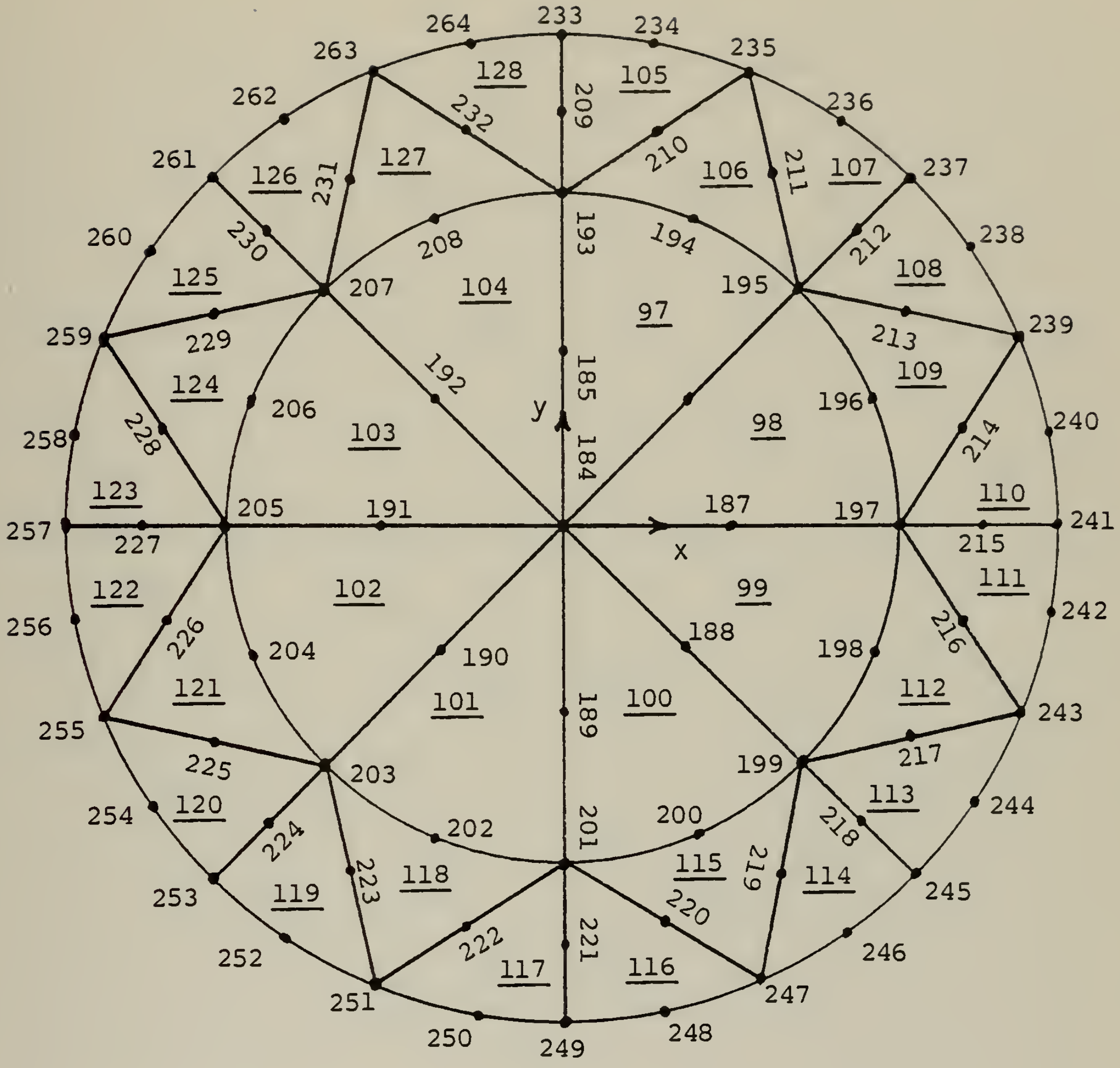
or system nodes corresponding to local or element nodes 1 through 6. The middle plane contained all the system nodes corresponding to the local nodes 7, 8, and 9. System nodes corresponding to element nodal points 10 through 15 comprised the bottom plane. To fix ideas, the three nodal planes of the first layer of mesh I are shown in figures 7, 8, and 9.

In this work there was only one curved side which was in the plane of the triangle, as shown in figure 10. The first, seventh, and tenth element nodes were each arbitrarily assigned to have as an opposite side the curved side of the triangle. The remaining local nodes in each of the respective planes of the element were then numbered consecutively in the counterclockwise direction.

At this point it is appropriate to define connectivity. The connectivity of an element is a row vector array that relates the local nodes to system nodal points. The connectivity lists the system nodal points that are "connected" to form the element domain in the sequence of local nodal numbering. Thus, the connectivity matrix for mesh I is a matrix of size 128 x 15. To illustrate, the connectivity of element number 106 is

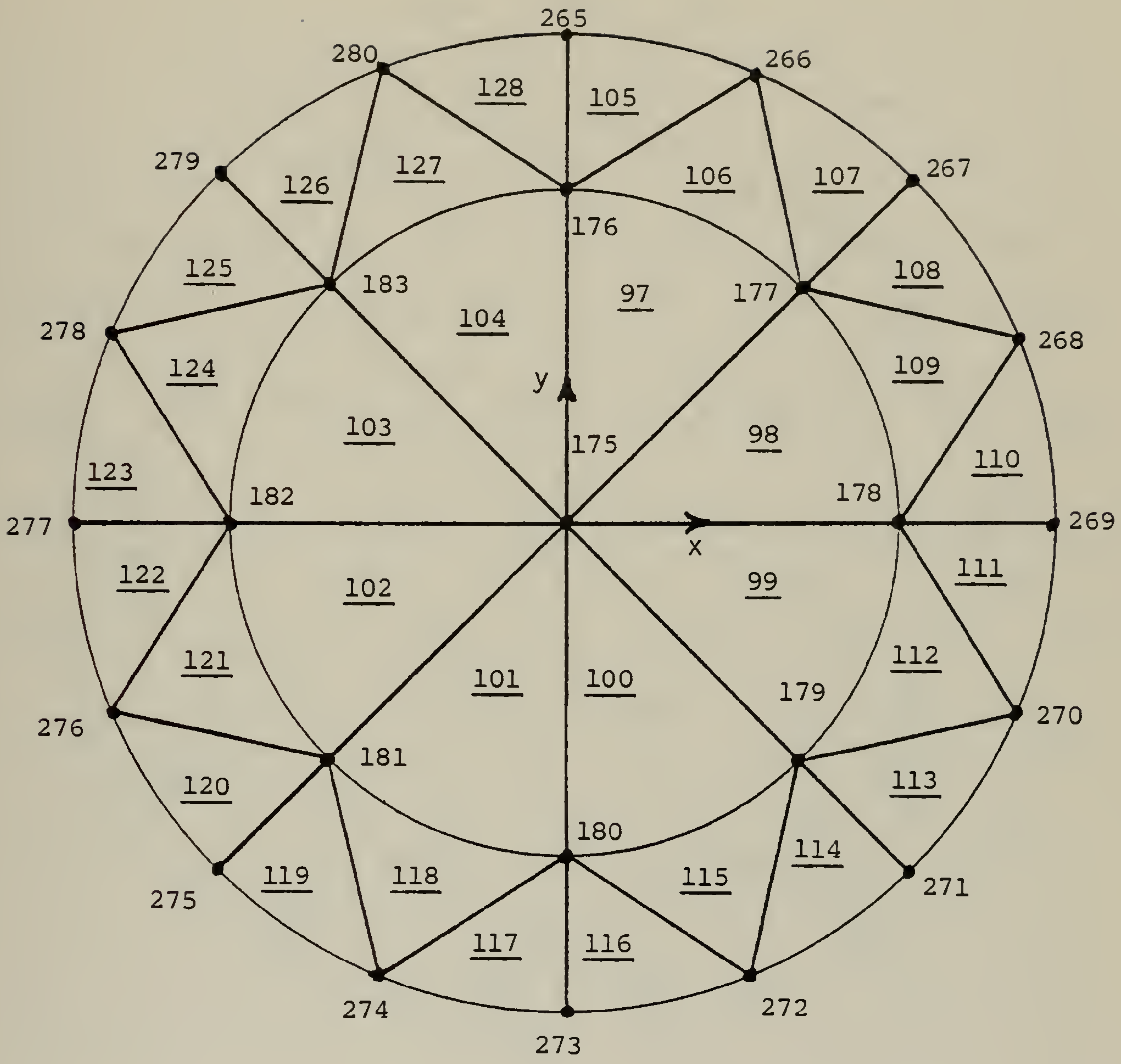
<235, 210, 193, 194, 195, 211, 266, 176, 177, 283, 152, 10, 11, 12, 53>

A second finite element mesh (mesh II) consisting of 192 elements was developed. It contained the same number of elements per layer as mesh I. However, mesh II has six layers. The second and third layers of mesh I were each divided in



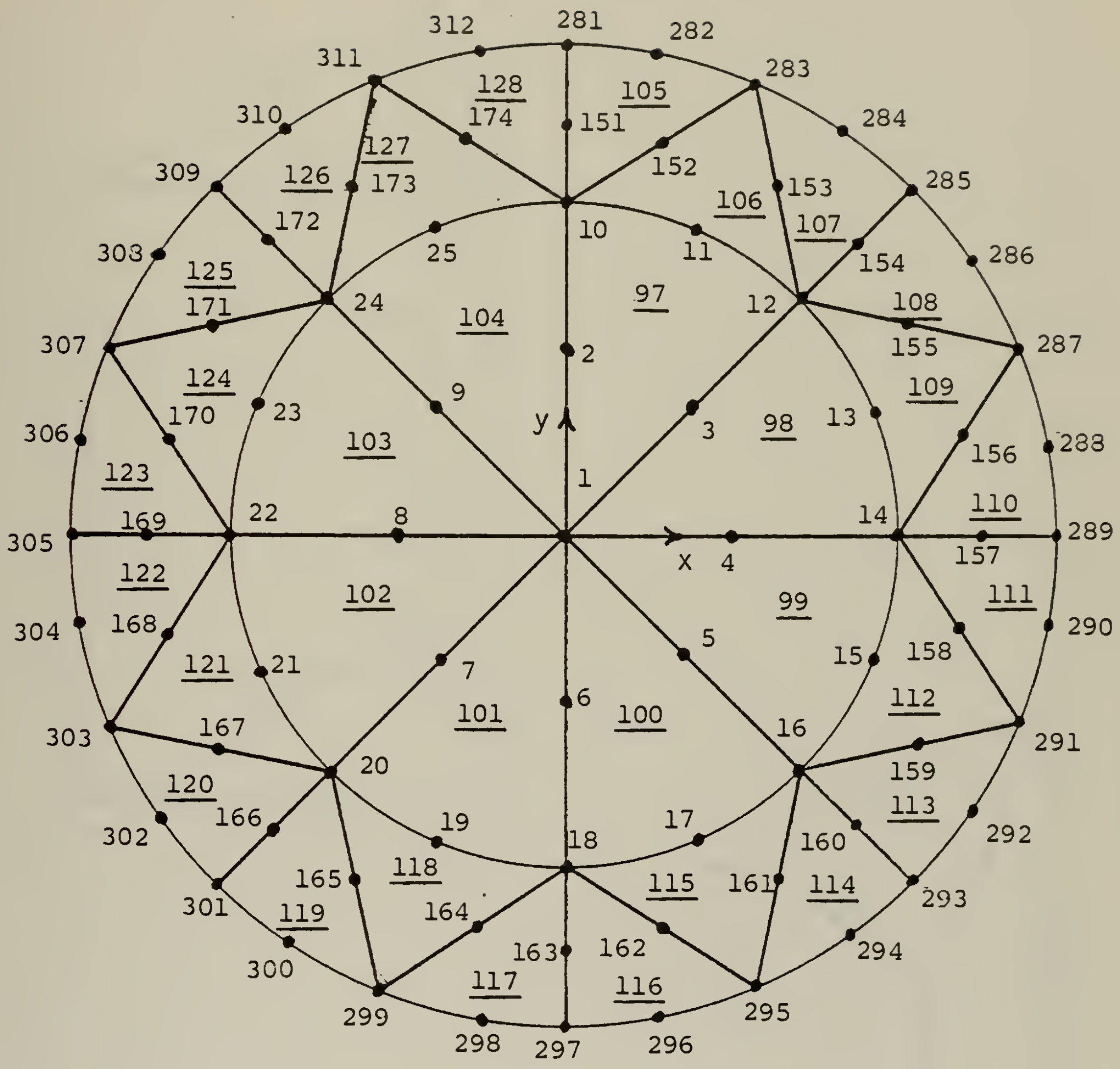
Number - element number

Figure 7. Top nodal plane of the first layer of mesh I, z = 110 cm



Number - element number

Figure 8. Middle nodal plane of the first layer of mesh I, z = 95 cm



Number - element number

Figure 9. Bottom nodal plane of the first layer of mesh I, z = 80 cm

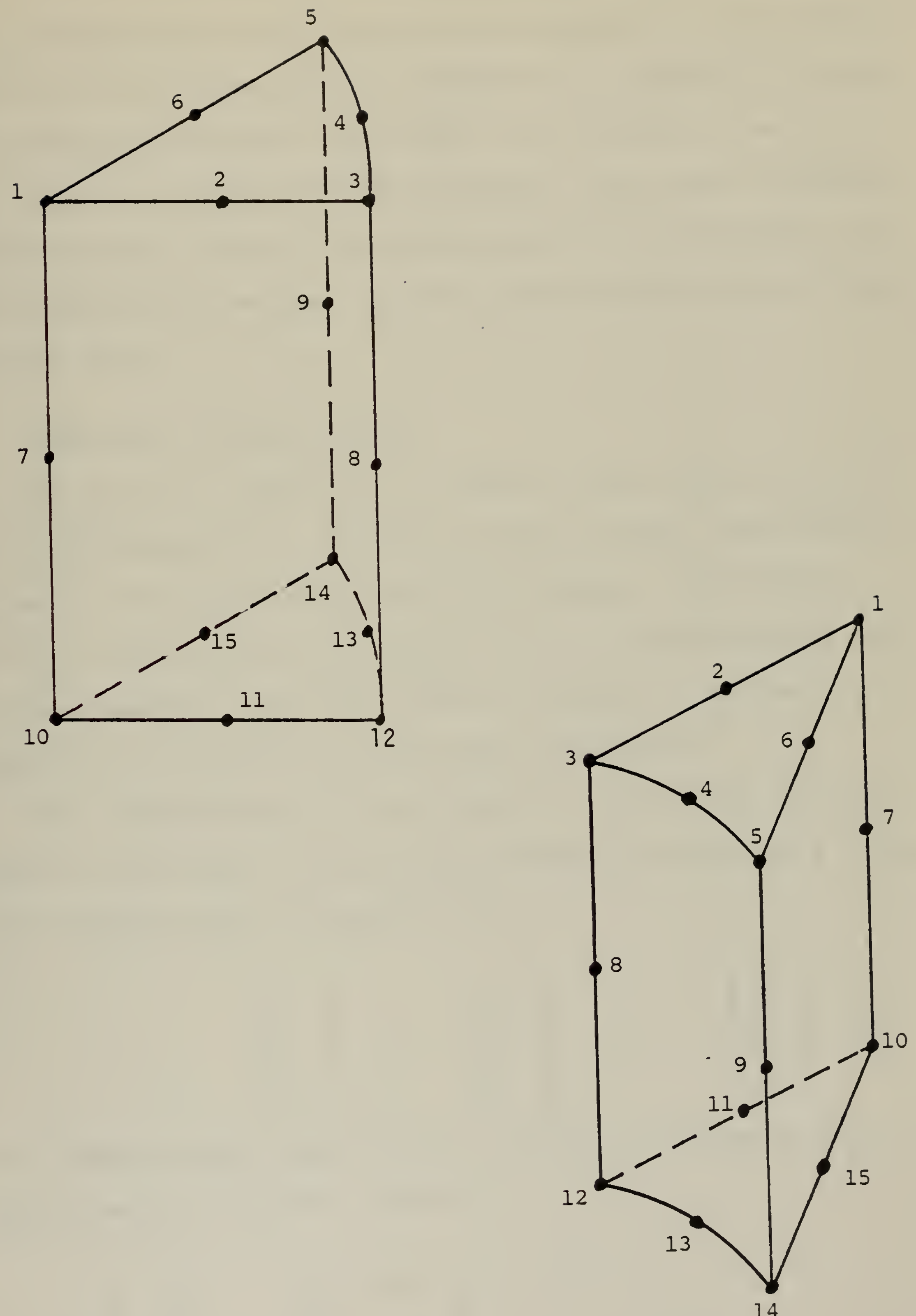


Figure 10. Local nodal numbering of curved elements

half, thus forming the two additional layers of mesh II. The connectivity matrix and the coordinates of each global node for mesh I are given in Appendix A. Appendix B lists the connectivity matrix and nodal coordinates of mesh II. The reader can construct the different layers and elements without much difficulty by using the nodal coordinates and the connectivity matrix. A mesh generator was not utilized in this work.

E. COORDINATE TRANSFORMATION

The use of a quadratic or higher order element permits the transformation or mapping of the straight-sided parent element into an element with curved sides. Distorted or curved elements provide a better fit to curved domains than linear elements, and thus a smaller number of elements is required to represent the structure adequately.

The transformation from cartesian coordinates to curvilinear coordinates can be accomplished by employing a one-to-one correspondence defined by [11]

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = f \begin{Bmatrix} \xi \\ \eta \\ \zeta \end{Bmatrix} \quad \text{or } f \begin{Bmatrix} L_1 \\ L_2 \\ L_3 \\ \zeta \end{Bmatrix} \qquad (46)$$

The element shape functions are utilized to achieve this transformation via the relation

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \begin{Bmatrix} \sum_i N_i x_i \\ \sum_i N_i y_i \\ \sum_i N_i z_i \end{Bmatrix} \quad , \quad i=1,2,\ldots n^e \qquad (47)$$

where n^e is the number of element nodes, and the element shape functions N_i are in terms of local coordinates. Each set of local coordinates corresponds to only one set of cartesian coordinates.

In performing the transformation, the compatibility requirement must be met. The transformation into the new, curved elements should leave no gaps between adjacent elements. If two adjacent elements are generated from parents in which the element shape functions satisfy continuity requirements, then the curved elements will be contiguous [11]. For the isoparametric element, uniqueness of coordinates ensures compatibility. Continuity is assured when adjacent elements are given the same sets of coordinates at common nodes.

Since the element shape functions are in terms of local coordinates, an element of volume, dxdydz, must be transformed into an element of volume expressed in local coordinates. This is achieved through the use of the Jacobian matrix defined below. Using the chain rule, the relationship between ξ,η,ζ and a corresponding set of cartesian coordinates x,y,z is

$$\begin{Bmatrix} \frac{\partial N_i}{\partial \xi} \\ \frac{\partial N_i}{\partial \eta} \\ \frac{\partial N_i}{\partial \zeta} \end{Bmatrix} = \begin{bmatrix} \frac{\partial x}{\partial \xi} , & \frac{\partial y}{\partial \xi} , & \frac{\partial z}{\partial \xi} \\ \frac{\partial x}{\partial \eta} , & \frac{\partial y}{\partial \eta} , & \frac{\partial z}{\partial \eta} \\ \frac{\partial x}{\partial \zeta} , & \frac{\partial y}{\partial \zeta} , & \frac{\partial z}{\partial \zeta} \end{bmatrix} \begin{Bmatrix} \frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial z} \end{Bmatrix} \qquad (48)$$

The Jacobian matrix [J] is defined as

$$[J] = \begin{bmatrix} \frac{\partial x}{\partial \xi} , \frac{\partial y}{\partial \xi} , \frac{\partial z}{\partial \xi} \\ \frac{\partial x}{\partial \eta} , \frac{\partial y}{\partial \eta} , \frac{\partial z}{\partial \eta} \\ \frac{\partial x}{\partial \zeta} , \frac{\partial y}{\partial \zeta} , \frac{\partial z}{\partial \zeta} \end{bmatrix} \qquad (49)$$

From equation (47), the Jacobian matrix becomes

$$[J] = \begin{bmatrix} \sum_i \frac{\partial N_i}{\partial \xi} x_i , \sum_i \frac{\partial N_i}{\partial \xi} y_i , \sum_i \frac{\partial N_i}{\partial \xi} z_i \\ \sum_i \frac{\partial N_i}{\partial \eta} x_i , \sum_i \frac{\partial N_i}{\partial \eta} y_i , \sum_i \frac{\partial N_i}{\partial \eta} z_i \\ \sum_i \frac{\partial N_i}{\partial \zeta} x_i , \sum_i \frac{\partial N_i}{\partial \zeta} y_i , \sum_i \frac{\partial N_i}{\partial \zeta} z_i \end{bmatrix} \qquad (50)$$

The determinant of the Jacobian is used to transform the volume of element in cartesian coordinates to local coordinates. For a volume of element [11],

$$dxdydz = \det [J] \, d\xi d\eta d\zeta \qquad (51)$$

Note that the determinant of the Jacobian is a variable for elements of curved geometry. Only in the case of straight-sided elements is the determinant of the Jacobian a constant.

In the plane of the triangle, the area coordinates (L_1, L_2,L_3) number one more than the cartesian coordinates (x,y). Thus, L_3 is defined as a dependent variable. This establishes the origin of the $\xi\eta$ coordinate system at corner point 3, as

illustrated in figure 11. Recall that the $\xi\eta$ axes need not be orthogonal. As such

$$\xi = L_1 \tag{52a}$$

$$\eta = L_2 \tag{52b}$$

Using equation (42),

$$L_3 = 1 - \xi - \eta \tag{52c}$$

Applying the chain rule yields

$$\frac{\partial N_i}{\partial \xi} = \frac{\partial N_i}{\partial L_1}\frac{\partial L_1}{\partial \xi} + \frac{\partial N_i}{\partial L_2}\frac{\partial L_2}{\partial \xi} + \frac{\partial N_i}{\partial L_3}\frac{\partial L_3}{\partial \xi} \tag{53a}$$

$$\frac{\partial N_i}{\partial \eta} = \frac{\partial N_i}{\partial L_1}\frac{\partial L_1}{\partial \eta} + \frac{\partial N_i}{\partial L_2}\frac{\partial L_2}{\partial \eta} + \frac{\partial N_i}{\partial L_3}\frac{\partial L_3}{\partial \eta} \tag{53b}$$

Using equations (52a), (52b), and (52c) in equations (53a) and (53b) gives

$$\frac{\partial N_i}{\partial \xi} = \frac{\partial N_i}{\partial L_1} - \frac{\partial N_i}{\partial L_3} \tag{54a}$$

$$\frac{\partial N_i}{\partial \eta} = \frac{\partial N_i}{\partial L_2} - \frac{\partial N_i}{\partial L_3} \tag{54b}$$

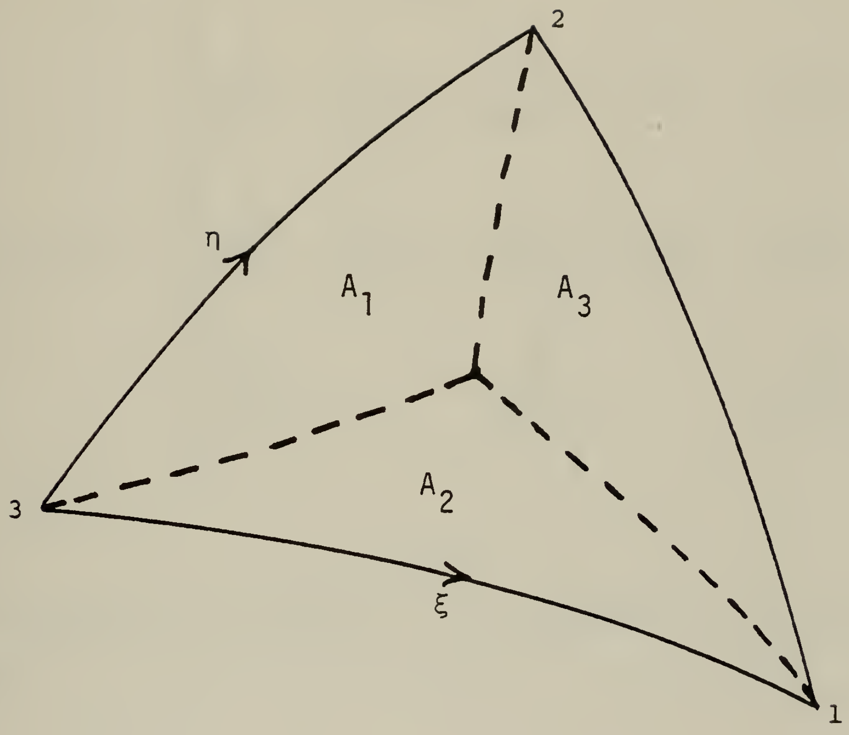


Figure 11. ηξ coordinates in a triangle

Now the Jacobian matrix can be evaluated using the shape functions $N_i = N_i(L_1, L_2, L_3, \zeta)$. Inserting equations (54a) and (54b) in equation (50) produces

$$[J(L_1 L_2, L_3, \zeta)] = \begin{bmatrix} \sum_i (\frac{\partial N_i}{\partial L_1} - \frac{\partial N_i}{\partial L_3}) x_i , & \sum_i (\frac{\partial N_i}{\partial L_1} - \frac{\partial N_i}{\partial L_3}) y_i , & \sum_i (\frac{\partial N_i}{\partial L_1} - \frac{\partial N_i}{\partial L_3}) z_i \\ \sum_i (\frac{\partial N_i}{\partial L_2} - \frac{\partial N_i}{\partial L_3}) x_i , & \sum_i (\frac{\partial N_i}{\partial L_2} - \frac{\partial N_i}{\partial L_3}) y_i , & \sum_i (\frac{\partial N_i}{\partial L_2} - \frac{\partial N_i}{\partial L_3}) z_i \\ \sum_i (\frac{\partial N_i}{\partial \zeta}) x_i , & \sum_i (\frac{\partial N_i}{\partial \zeta}) y_i , & \sum_i (\frac{\partial N_i}{\partial \zeta}) z_i \end{bmatrix} \tag{55}$$

To summarize, suppose it is required to transform the integral

$$I = \int_{Vol} F'(L_1, L_2, L_3, \zeta) \, dxdydz \tag{56}$$

to an integral entirely in terms of local coordinates. The determinant of equation (55) is then utilized to give

$$I = \int_{-1}^{1} \int_{0}^{1} \int_{0}^{1-L_1} F'(L_1, L_2, L_3, \zeta) \det[J(L_1, L_2, L_3, \zeta)] dL_1 dL_2 d\zeta \tag{57}$$

Equation (56) is now in a form suitable for numerical integration.

The development of coordinate transformation to this point has been under the general assumption that all the sides of the element are curved. In this work, the only

curved side is in the plane of the triangle. The sides of the element along the prism axis are straight. As such, a modified Jacobian matrix can be employed, thereby reducing the number of calculations to be performed. This modified Jacobian is the 2x2 matrix defined by

$$[J^*] = \begin{bmatrix} \sum_i (\frac{\partial N_i}{\partial L_1} - \frac{\partial N_i}{\partial L_3}) x_i \ , & \sum_i (\frac{\partial N_i}{\partial L_1} - \frac{\partial N_i}{\partial L_3}) y_i \\ \sum_i (\frac{\partial N_i}{\partial L_2} - \frac{\partial N_i}{\partial L_3}) x_i \ , & \sum_i (\frac{\partial N_i}{\partial L_1} - \frac{\partial N_i}{\partial L_3}) y_i \end{bmatrix} \qquad (58)$$

Along the prism axis or ζ direction, it can be shown that

$$dz = \frac{h}{2} d\zeta \qquad (59)$$

where h = height of the element. The volume relationship is then given by

$$dxdydz = \frac{h}{2} \det[J^*] \ dL_1 dL_2 \zeta \qquad (60)$$

The integral of equation (56) then assumes the form of

$$I = \frac{h}{2} \int_{-1}^{1} \int_{0}^{1} \int_{0}^{1-L_1} F'(L_1, L_2, L_3, \zeta) \det[J^*(L_1, L_2, L_3, \zeta)] dL_1, dL_2, d\zeta \qquad (61)$$

Equation (61) is the basis of numerical integration applied in this work.

F. CONSTRUCTION OF ELEMENT MATRICES

The system matrix operators can be constructed through the use of the global basis functions G_J or through the

application of the element shape functions. Although the global basis functions were used to demonstrate the method of Galerkin, the construction of the system matrices in this work was achieved through element considerations. The solution of the unknown variable ψ within the element domain was approximated by

$$\tilde{\psi}^e = \sum_{i=1} N_i \, \psi_i^e(t) \ , \quad i=1,2,\ldots,n^e \tag{62}$$

where n^e is the number of element nodal points, N_i are the element shape functions, and $\psi_i^e(t)$ are the time-dependent nodal magnitudes of $\tilde{\psi}^e$. The element contribution to the system matrix operators is defined by Galerkin's orthogonality condition expressed by

$$\int_{Vol} N_j \, R^e \, dVol = 0 \ , \ j=1,2,\ldots,n^e \tag{63}$$

where R^e is defined by replacing $\tilde{\psi}$ with $\tilde{\psi}^e$ in equations (37) and (38), and the integration is over the element volume. Using equation (62), the element contribution is portrayed for the core by

$$\begin{aligned}&[\int_{Vol} N_j N_i dVol]\{\dot{\psi}_i{}^e(t)\} - [\int_{Vol} N_j \nabla^2 N_i dVol]\{(c1\cdot\psi^e)_i\} + [\int_{Vol} N_j N_i dVol]\{(c2\cdot\psi^e)_i\}\\ &+ [\int_{Vol} N_j N_i \ell n(1 + \frac{K}{\gamma T_o} \sum_k N_k \, \psi_k{}^e(t))dVol] \, \{(c4\cdot\psi^e)_i\}\\ &+ [\int_{Vol} N_j N_i dVol] \, \{\int_0^t e^{-\bar{\lambda}(t-t')} \, (c5\cdot\psi(t'))_i dt'\} = 0 \end{aligned} \tag{64}$$

where $i,j,k = 1,2,\ldots,15$, and c1, c2, etc., are constants at node i. The bracketed expressions represent square matrices of size 15x15, and the braced expressions represent column vectors of size 15x1. For the reflector, the last two terms of equation (64) are zero. Before any operation can be performed on equation (64), the nonlinear terms (last two terms) must be "linearized" and the term with the ∇^2 operator must be integrated by parts.

The nonlinear feedback term, $\ell n[1 + \frac{K}{\gamma T_o} \sum_{k=1}^{15} N_k \psi_k{}^e(t)]$, was linearized by using predicted values of the unknown function at time t. These predicted values come from the time integration scheme by Franke [7]. Basically, the integration scheme utilizes a predictor-corrector method which predicts values of the unknown function at the next time by using the derivatives of the function. Adopting the predicted values $\psi_k{}^P$ enabled integration over space. The term in equation (64) involving the nonlinear feedback can be written as

$$[\int_{Vol} N_j N_i \ell n(1 + \frac{K}{\gamma T_o}(N_1\psi_1{}^P + N_2\psi_2{}^P + \ldots + N_{15}\psi_{15}{}^P))dVol] \{(c4\cdot\psi^e)_i\}$$

The last tern of equation (64) describes the delayed neutron contribution. In general,

$$e^{-\bar{\lambda} t_n} \int_0^{t_n} e^{\bar{\lambda} t'} \psi_i{}^e(t')dt' = e^{-\bar{\lambda}(t_n - t_{n-1})} [e^{-\bar{\lambda} t_{n-1}} \int_0^{t_{n-1}} e^{\bar{\lambda} t'} \psi_i{}^e(t')dt'] + e^{-\bar{\lambda} t_n} \int_{t_{n-1}}^{t_n} e^{\bar{\lambda} t'} \psi_i{}^e(t')dt' \qquad (65)$$

where n is number of time steps, t_n is the current time,

and t_{n-1} is the previous time. To approximate the integrals in equation (65), the predicted values ψ_i^P were employed in a simple trapezoidal rule. The trapezoidal rule was believed to be sufficient since very small time steps were utilized in this work. For example, at time t_2 ,

$$(\mathrm{SUM}_i) = (\text{previous } \mathrm{SUM}_i)e^{-\bar{\lambda}H} + \frac{H}{2}(e^{\bar{\lambda}H}\psi_i^e(t_1) + \psi_i^P)$$

where

$$H = \text{current time step} = t_2 - t_1$$

$$(\mathrm{SUM}_i) = e^{-\bar{\lambda}t_2} \int_o^{t_2} e^{\bar{\lambda}t'} \psi_i^e(t')dt'$$

$$(\text{previous } \mathrm{SUM}_i) = e^{-\bar{\lambda}t_1} \int_o^{t_1} e^{\bar{\lambda}t'} \psi_i^e(t')dt'$$

The previous sum is formed by accumulating the trapezoidal integration of each time step. In general then, for time t_n

$$(\mathrm{SUM}_i) = (\text{previous } \mathrm{SUM}_i)e^{-\bar{\lambda}H} + \frac{H}{2}(e^{\bar{\lambda}H}\psi_i^e(t_{n-1}) + \psi_i^P) \quad (66)$$

The delayed neutron term in equation (64) can be expressed as

$$[\int_{\mathrm{Vol}} N_j N_i d\mathrm{Vol}]\ \{(c5 \cdot \mathrm{SUM})_i\}$$

In order to bring equation (64) to final form, the ∇^2 operator was integrated by parts. Since the flux at the surface of the reactor is zero, integration by parts yields

$$\int_{Vol} N_j \nabla^2 N_i dxdydz = - \int_{Vol} \left(\frac{\partial N_i}{\partial x}\frac{\partial N_j}{\partial x} + \frac{\partial N_i}{\partial y}\frac{\partial N_j}{\partial y} + \frac{\partial N_i}{\partial z}\frac{\partial N_j}{\partial z}\right) dxdydz \qquad (67)$$

In vector notation, equation (67) is expressed as

$$\int_{Vol} N_j \nabla^2 N_i dxdydz = - \int_{Vol} \left< \frac{\partial N_i}{\partial x} , \frac{\partial N_i}{\partial y} , \frac{\partial N_i}{\partial z} \right> \begin{Bmatrix} \frac{\partial N_j}{\partial x} \\ \frac{\partial N_j}{\partial y} \\ \frac{\partial N_j}{\partial z} \end{Bmatrix} dxdydz \qquad (68)$$

Using the chain rule produces

$$\begin{Bmatrix} \frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial z} \end{Bmatrix} = \begin{bmatrix} \frac{\partial L_1}{\partial x} , & \frac{\partial L_2}{\partial x} , & 0 \\ \frac{\partial L_1}{\partial y} , & \frac{\partial L_2}{\partial y} , & 0 \\ 0 , & 0 , & \frac{\partial \zeta}{\partial z} \end{bmatrix} \begin{Bmatrix} (\frac{\partial N_i}{\partial L_1} - \frac{\partial N_i}{\partial L_3}) \\ (\frac{\partial N_i}{\partial L_2} - \frac{\partial N_i}{\partial L_3}) \\ (\frac{\partial N_i}{\partial \zeta}) \end{Bmatrix} \qquad (69)$$

Letting the 3x3 matrix above be [B'], the ∇^2 term becomes

$$\int_{Vol} N_j \nabla^2 N_i dxdydz = - \int_{Vol} \left< (\frac{\partial N_i}{\partial L_1} - \frac{\partial N_i}{\partial L_3}) , (\frac{\partial N_i}{\partial L_2} - \frac{\partial N_i}{\partial L_3}) , \frac{\partial N_i}{\partial \zeta} \right> [B']^T [B'] \begin{Bmatrix} (\frac{\partial N_j}{\partial L_1} - \frac{\partial N_j}{\partial L_3}) \\ (\frac{\partial N_j}{\partial L_2} - \frac{\partial N_j}{\partial L_3}) \\ \frac{\partial N_j}{\partial \zeta} \end{Bmatrix} dxdydz \qquad (70)$$

where $[B']^T$ is the transpose of $[B']$. By applying the chain rule in equation (47) and using equation (59), it can be shown that for this work

$$[B'] = \begin{bmatrix} \dfrac{1}{(\frac{\partial N_1}{\partial L_1} x_1 + \ldots + \frac{\partial N_{15}}{\partial L_1} x_{15})} , & \dfrac{1}{(\frac{\partial N_1}{\partial L_2} x_1 + \ldots + \frac{\partial N_{15}}{\partial L_2} x_{15})} , & 0 \\ \dfrac{1}{(\frac{\partial N_1}{\partial L_1} y_1 + \ldots + \frac{\partial N_{15}}{\partial L_1} y_{15})} , & \dfrac{1}{(\frac{\partial N_1}{\partial L_2} y_1 + \ldots + \frac{\partial N_{15}}{\partial L_2} y_{15})} , & 0 \\ 0 , & 0 , & \frac{2}{h} \end{bmatrix} \quad (71)$$

$[B']^T$ can be derived from equation (71).

Note from equation (64) that there are three basic element matrices which are defined as follows after applying equation (60):

$$[G_{ji}] = \frac{h}{2} \int_{-1}^{1}\int_{o}^{1}\int_{o}^{1-L_1} N_j N_i \det[J^*] dL_1 dL_2 d\zeta \quad (72)$$

$$[GG_{ji}] = \frac{h}{2} \int_{-1}^{1}\int_{o}^{1}\int_{o}^{1-L_1} < \frac{\partial N_i}{\partial L_1} - \frac{\partial N_i}{\partial L_3}) , (\frac{\partial N_i}{\partial L_2} - \frac{\partial N_i}{\partial L_3}) , (\frac{\partial N_i}{\partial \zeta}) > [B']^T$$

$$[B'] \left\{ \begin{matrix} (\frac{\partial N_j}{\partial L_1} - \frac{\partial N_j}{\partial L_3}) \\ (\frac{\partial N_j}{\partial L_2} - \frac{\partial N_j}{\partial L_3}) \\ \frac{\partial N_j}{\partial \zeta} \end{matrix} \right\} \det[J^*] dL_1 dL_2 d\zeta \quad (73)$$

$$[GGG_{ji}] = \frac{h}{2} \int_{-1}^{1}\int_{o}^{1}\int_{o}^{1-L_1} N_j N_i \ell n(1+ \frac{K}{\gamma T_o}(N_1 \psi_1{}^P + \ldots + N_{15} \psi_{15}^P)) \det[J^*] dL_1 dL_2 d\zeta$$

$$(74)$$

Element matrices $[G_{ji}]$ and $[GG_{ji}]$ are independent of time. However, $[GGG_{ji}]$ is time dependent due to the utilization of the predicted values $\psi_i{}^p$ which changes with time. In terms of these three basic element matrices, equation (64) is

$$[G_{ji}]\{\dot{\psi}_i^e\} + [GG_{ji}]\{c1\cdot\psi^e)_i\} + [G_{ji}]\{(c2\cdot\psi^e)_i\}$$

$$+ [GGG_{ji}]\ \{(c4\cdot\psi^e)_i\} + [G_{ji}]\ \{(c5\cdot SUM)_i\} \ = 0 \qquad (75)$$

The last two terms of equation (75) are zero for the reflector.

G. CONSTRUCTION OF THE SYSTEM MATRICES

The 15x15 coefficient element matrices were calculated according to equations (72), (73), and (74); and the results were collected element by element into the corresponding system coefficient matrices. The system coefficient matrix [BIGG] is developed from $[G_{ji}]$, [BIGGG] from $[GG_{ji}]$, and [BIGH] from $[GGG_{ji}]$. [BIGG] and [BIGGG] are independent of time and can be constructed once and for all from geometry considerations. [BIGH] is dependent on both geometry and time due to the time dependence of the predicted flux utilized in the feedback term. Thus, [BIGH] is recalculated at each time increment.

Non-zero contributions to a global nodal point I come only from adjacent elements sharing that same nodal point I. Thus, the system matrices are sparse and banded. The process of assembling contributions from element matrices

requires the identification of a local nodal point (i=1,2, ...,15) with a global nodal point (I=1,2,...NUMNP, where NUMNP is the total number of system nodes). This correspondence between element and global nodes is accomplished via the connectivity matrix.

The formal treatment of the field equations in terms of the system coefficient matrices is described by the equation

$$[BIGG]\ \{\dot{\psi}_I\} + [BIGGG]\ \{(c1\cdot\psi)_I\} + [BIGG]\ \{(c2\cdot\psi)_I\}$$
$$+ [BIGH]\ \{(c4\cdot\psi)_I\} + [BIGG]\ \{(c5\cdot SUM)_I\} = 0 \qquad (76)$$

where the system matrices are NUMNP x NUMNP and the column vectors are of length NUMNP x 1. However, the direct application of equation (76) requires a large amount of computer storage. To take advantage of the sparsity of the system matrices, an optimum compacting scheme described by Ref. 8 was employed.

The concept behind OCS is simply to store only the non-zero terms of a coefficient matrix. OCS requires two integer arrays, say JB and NAME, and a vector of non-zero coefficients of the square system matrix. For purposes of illustration, the square system matrix is called B, and the vector of non-zero coefficients of B is called BB. The ith integer entry in the NUMNP x 1 JB vector is the number q_i. This number is defined by

$$q_i = 1 + \sum_{j=1}^{i=1} p_j \ , \ i=1,2,...,NUMNP \qquad (77)$$

where p_j is the number of terms in the ith equation. In

other words, p_j is the number of nodes that the i^{th} node "sees". JB is therefore a pointer vector of length NUMNP+1 whose i^{th} term locates the initial position in the BB vector of the contributing coefficients to the i^{th} equation. The NAME vector of length Mx1, where $M = \sum_{i=1}^{NUMNP} p_i$, consists of NUMNP successive vector blocks of variable length p_i, i=1,2,...,NUMNP. The p_i integer numbers in the i^{th} block of NAME list the p_i contributors to the i^{th} equation. The Mx1 BB vector contains the real non-zero coefficients of the NUMNPxNUMNP B matrix, arranged in the same contiguous block arrangement as the NAME vector. The j^{th} term in the i^{th} block or BB(JB(I) + J-1) is B(I,K), where K = NAME(JB(I)+J-1). To illustrate, consider the grid shown in figure 12. The two array vectors and the coefficient vector matrix of non-zero terms are:

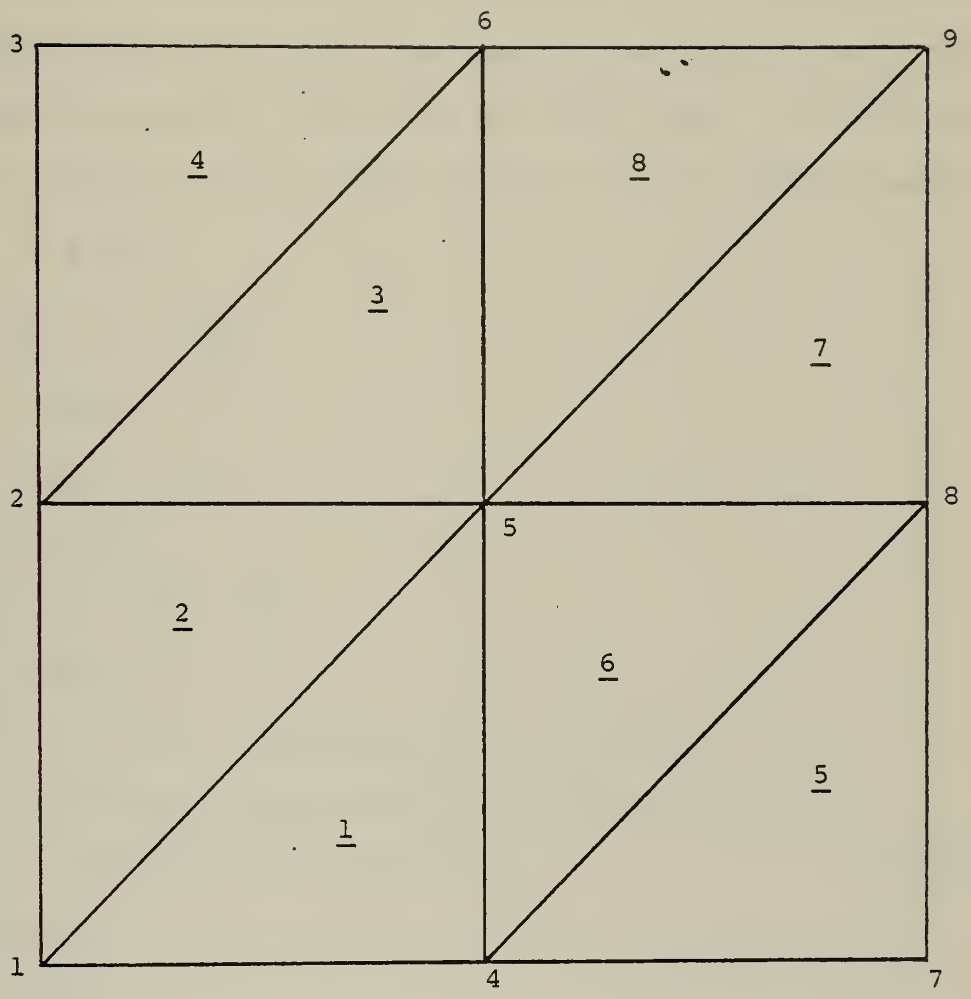
JB = <1, 5, 10, 13, 18, 25, 30, 33, 38, 42>

NAME = <1,2,4,5|2,3,1,6,5| --- |9,8,6,5>

BB = $<B_{11}, B_{12}, B_{14}, B_{15} | B_{22}, B_{23}, B_{21}, B_{26}, B_{25} | --- | B_{99}, B_{98}, B_{96}, B_{95}>$

In this illustration, NUMNP = 9 and M = 41 [8].

In this work, a judicious method of numbering the system nodal points was adopted to further reduce computer storage requirements. Since the surface boundary nodes of the reactor represent zero neutron fluxes, the contributions of these nodes to interior or non-zero nodes can be discarded. Thus, only the interior nodal points need to be considered. These non-zero nodes were numbered first in the finite element mesh



Number - element number

Figure 12. Sample grid used for illustrating OCS

used so that in the OCS, the number of non-zero nodes (NNZ) replaces NUMNP.

The vectors of non-zero coefficients will be designated BIGG, BIGGG and BIGH since the square coefficient matrices described in equation (76) were not utilized. To illustrate the application of OCS in the system, the following sample program is given:

```
    DO  450  I=1, NNZ
    JBB = JB(I)
    JE = JB(I+1)-1
    DY(I) = 0.0
    DO  500  J=JBB, JE
    LL = NAME(J)
    DY(I) = DY(I) + BIGG(J)*ψ̇(LL) + BIGGG(J)*c1(LL)*
    ψ(LL) + BIGG(J)*c2(LL)*ψ(LL) + BIGH(J)*c4(LL)*
    ψ(LL) + BIGG(J)*c5(LL)*SUM(LL)                      (78)
 500 CONTINUE
 450 CONTINUE
```

where

LL = nodal point "seen" by node I

JE-JBB = total number of nodal points that node I "sees"

DY(I) = summation of all contributions to node I and should sum to zero as stated by the field equation

The assumption of a homogeneous reflector was relaxed in the application of equation (78). Interface nodes were assigned properties of the core. Therefore, if LL is a reflector

node not on the core reflector interface, the last two terms of equation (78) are nonexistent.

IV. NUMERICAL INTEGRATION

A. LINE AND AREA INTEGRATION

The solution of the element matrix equations was achieved through numerical integration since an exact closed form solution cannot be established. The volume integration was accomplished by using a line integration in the ζ-direction and an area integration in the plane of the triangle. The line integral is described by the Gaussian quadrature formula [12]

$$\int_{-1}^{1} f(\zeta)d\zeta \approx \sum_{k=1}^{n} H_k f(a_k) \qquad (79)$$

where n is the number of Gauss integration points

H_k = weighting coefficients

$f(a_k)$ = the function $f(\zeta)$ evaluated at Gauss point a_R

Table III lists a_k, H_k and n [11].

The area integration was achieved by the equation

$$\int_0^1 \int_0^{1-L_1} f(L_1,L_2,L_3)dL_1 dL_2 \approx \sum_{m=1}^{\bar{m}} w_m f(L_1{}^m, L_2{}^m, L_3{}^m) \qquad (80)$$

where $\bar{m}$ is the number of area integration points and w_m are the weights. The numerical integration points for the area integration are given in Table IV which was extracted

TABLE III

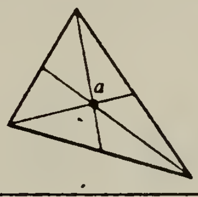
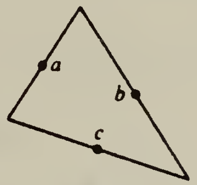
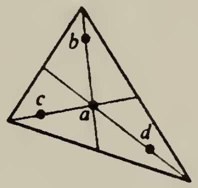
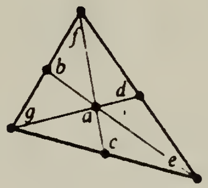
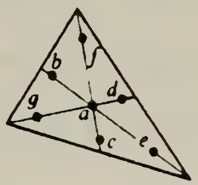
Abscissae and Weight Coefficients of the Gaussian Quadrature Formula

$$\int_{-1}^{1} f(\zeta)d\zeta = \sum_{k=1}^{n} H_k f(a_k)$$

$\pm a$		H
	$n = 2$	
0·57735 02691 89626		1·00000 00000 00000
	$n = 3$	
0·77459 66692 41483		0·55555 55555 55556
0·00000 00000 00000		0·88888 88888 88889
	$n = 4$	
0·86113 63115 94053		0·34785 48451 37454
0·33998 10435 84856		0·65214 51548 62546
	$n = 5$	
0·90617 98459 38664		0·23692 68850 56189
0·53846 93101 05683		0·47862 86704 99366
0·00000 00000 00000		0·56888 88888 88889
	$n = 6$	
0·93246 95142 03152		0·17132 44923 79170
0·66120 93864 66265		0·36076 15730 48139
0·23861 91860 83197		0·46791 39345 72691
	$n = 7$	
0·94910 79123 42759		0·12948 49661 68870
0·74153 11855 99394		0·27970 53914 89277
0·40584 51513 77397		0·38183 00505 05119
0·00000 00000 00000		0·41795 91836 73469
	$n = 8$	
0·96028 98564 97536		0·10122 85362 90376
0·79666 64774 13627		0·22238 10344 53374
0·52553 24099 16329		0·31370 66458 77887
0·18343 46424 95650		0·36268 37833 78362
	$n = 9$	
0·96816 02395 07626		0·08127 43883 61574
0·83603 11073 26636		0·18064 81606 94857
0·61337 14327 00590		0·26061 06964 02935
0·32425 34234 03809		0·31234 70770 40003
0·00000 00000 00000		0·33023 93550 01260
	$n = 10$	
0·97390 65285 17172		0·06667 13443 08688
0·86506 33666 88985		0·14945 13491 50581
0·67940 95682 99024		0·21908 63625 15982
0·43339 53941 29247		0·26926 67193 09996
0·14887 43389 81631		0·29552 42247 14753

TABLE IV

Numerical Formulas for Triangles

Order	Fig.	Error	Points	Triangular Co-ordinates	Weights $2W_i$
Linear		$R = 0(h^2)$	a	$\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$	1
Quadratic		$R = 0(h^3)$	a	$\frac{1}{2}, \frac{1}{2}, 0$	$\frac{1}{3}$
			b	$0, \frac{1}{2}, \frac{1}{2}$	$\frac{1}{3}$
			c	$\frac{1}{2}, 0, \frac{1}{2}$	$\frac{1}{3}$
Cubic		$R = 0(h^4)$	a	$\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$	$-\frac{27}{48}$
			b	$\frac{11}{15}, \frac{2}{15}, \frac{2}{15}$	$\frac{25}{48}$
			c	$\frac{2}{15}, \frac{11}{15}, \frac{2}{15}$	
			d	$\frac{2}{15}, \frac{2}{15}, \frac{11}{15}$	
Cubic		$R = 0(h^4)$	a	$\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$	$\frac{27}{60}$
			b	$\frac{1}{2}, \frac{1}{2}, 0$	$\frac{8}{60}$
			c	$0, \frac{1}{2}, \frac{1}{2}$	
			d	$\frac{1}{2}, 0, \frac{1}{2}$	
			e	$1, 0, 0$	$\frac{3}{60}$
			f	$0, 1, 0$	
			g	$0, 0, 1$	
Quintic		$R = 0(h^6)$	a	$\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$	0·225
			b	$\alpha_1, \beta_1, \beta_1$	0·13239415
			c	$\beta_1, \alpha_1, \beta_1$	
			d	$\beta_1, \beta_1, \alpha_1$	
			e	$\alpha_2, \beta_2, \beta_2$	0·12593918
			f	$\beta_2, \alpha_2, \beta_2$	
			g	$\beta_2, \beta_2, \alpha_2$	

with

$\alpha_1 = 0{\cdot}05971587$

$\beta_1 = 0{\cdot}47014206$

$\alpha_2 = 0{\cdot}79742699$

$\beta_2 = 0{\cdot}10128651$

from Ref. 11. The volume integration of the function $f(L_1,L_2,L_3,\zeta)$ using numerical integration is therefore

$$\int_{-1}^{1}\int_{0}^{1}\int_{0}^{1-L_1} f(L_1,L_2,L_3,\zeta)dL_1dL_2d\zeta \approx$$

$$\sum_{k=1}^{\bar{n}} w_k \sum_{m=1}^{\bar{m}} w_m f(L_1{}^m,L_2{}^m,L_3{}^m,\zeta^k) \qquad (81)$$

where $w_k = H_k$. In the manner of equation (81), the element matrices become

$$[G_{ji}] = \frac{h}{2} \sum_{k=1}^{\bar{n}} w_k \sum_{m=1}^{\bar{m}} w_m \; N_j(L_1{}^m,L_2{}^m,L_3{}^m,\zeta^k)N_i\det[J^*] \qquad (82)$$

$$[GG_{ji}] = \frac{h}{2} \sum_{k=1}^{\bar{n}} w_k \sum_{m=1}^{\bar{m}} w_m \; F\;(L_1{}^m,L_2{}^m,L_3{}^m,\zeta^k) \;\; \det[J^*] \qquad (83)$$

$$[GGG_{ji}] = \frac{h}{2} \sum_{k=1}^{\bar{n}} w_k \sum_{m=1}^{\bar{m}} w_m \; T\;(L_1{}^m,L_2{}^m,L_3{}^m,\zeta^k) \;\; \det[J^*] \qquad (84)$$

where F and T are easily derived from equations (73) and (74), respectively. Note that N_i and $\det[J^*]$ are also evaluated at each integration point. They are not shown as such merely for the sake of convenience in writing the equations.

B. NUMBER OF INTEGRATION POINTS

It is difficult to estimate the number of integration points required for good accuracy due to the complexity of the functions involved. The basic rule that the best number of integration points is found by trial and experience was adopted. For the $[G_{ji}]$ element matrix, the numbers involved

can be approximated. This was done by letting the determinant of the Jacobian equal to twice the area of the curved triangle (checking det[J*] at each integration point showed that this assumption was not too unreasonable). With det[J*] outside the integration process, $[G_{ji}]$ can be solved in closed form by integrating out ζ from -1 to +1 and then applying the closed form equation [12]

$$\int_{Area} L_1^{m_1} L_2^{m_2} L_3^{m_3} \, d(Area) = 2A \frac{m_1!m_2!m_3!}{(m_1+m_2+m_3+2)!} \tag{85}$$

where m_1, m_2, m_3 are positive integer exponents and A is the area of the triangle.

Five test points within the 15x15 element matrix were selected, as listed in Table V. The values obtained from the application of equation (85) are also given in Table V. Three sets of area integration points, each with a different number of ζ Gauss points, were used. These three sets of area integration points, given in Table IV, are:

1. cubic order (4 points)
2. cubic order (7 points)
3. quintic order (7 points)

Using each of the three integration points above with different ζ Gauss points in equation (82) produced the results obtained in Table V. From these results, the quintic order area integration points were selected for the element matrix $[G_{ji}]$ with three ζ Gauss points in the prism axes. This set of integration points was also used for $[GGG_{ji}]$.

TABLE V.

Selection of Integration Points for $[G_{ji}]$

ς points	Area points	G(1,1)	G(2,2)	G(1,9)	G(6,9)	G(9,9)
13	cubic (4 pts)	1415	5278	-2756	5072	8536
5	"	1817	5278	-3170	5072	10240
7	"	1817	5278	-3170	5072	10240
3	cubic (7 pts)	3248	8247	-3114	4960	10243
5	"	3249	8247	-3114	4960	10240
7	"	3249	8247	-3114	4960	10240
3	quintic (7 pts)	2464	6600	-3144	5020	10240
5	"	2464	6600	-3144	5020	10240
7	"	2464	6600	-3144	5020	10240
Approximated values		2500	6700	-3142	5026	10053

The numbers for the $[GG_{ji}]$ element matrix could not be approximated. The best that could be done was to obtain an idea of the order of magnitude of this element matrix. Towards this end, a linear triangular element was assumed. Using linear approximation, the order of magnitude was found to be about 10^3. Using the three different area integration points mentioned above with varying ζ Gauss points in equation (83) yielded the results given in Table VI. Further checks on the $[GG_{ji}]$ element matrix showed that the cubic order with four area integration points yielded the desired order of magnitude. Thus, the fourth order cubic with five ζ Gauss points was employed for the $[GG_{ji}]$ element matrix. A note should be mentioned here in regards to the vast difference in results obtained for $[GG_{ji}]$ using different area integration points. Most likely, it was due to the $[B']$ and $[B']^T$ matrices which required the inversion of $\frac{\partial x}{\partial L_1}$, $\frac{\partial x}{\partial L_2}$, etc. Or perhaps it was caused by the nature of the hybrid element used in this work. In any case, further investigation is warranted in this area.

TABLE VI

Selection of Integration Points for [GG$_{ji}$]

ζ points	Area points	GG(1,9)	GG(3,3)	GG(14,9)	GG(8,8)	GG(15,15)
3	cubic (4 pts)	18.4	170.1	-165	165.9	106.1
5	"	22.0	177.6	-186	195.9	106.1
7	"	22.0	177.6	-186	195.9	106.1
3	cubic (7 pts)	-1.03x10^{4}	2.11x10^{7}	1.05x10^{7}	8.42x10^{7}	2056
5	"	-1.03x10^{4}	2.11x10^{7}	1.05x10^{7}	8.42x10^{7}	2056
7	"	-1.03x10^{4}	2.11x10^{7}	1.05x10^{7}	8.41x10^{7}	2056
3	quintic (7 pts)	-52.6	1.27x10^{4}	-6.64x10^{4}	6.84x10^{4}	1.51x10^{5}
5	"	-52.6	1.27x10^{4}	-6.64x10^{4}	6.84x10^{4}	1.51x10^{5}
7	"	-52.6	1.27x10^{4}	-6.64x10^{4}	6.84x10^{4}	1.51x10^{5}

V. TEST PROBLEMS AND RESULTS

The reactor was subjected to uniform and local perturbations in the form of a ramp input described by

$$\Sigma_f = \Sigma_f^* + \alpha t \tag{86}$$

where α is the change in Σ_f per unit time and Σ_f^* is the critical fission cross-section. Σ_f^* must first be obtained before the perturbations can be applied. This was accomplished by trial and error until a stationary solution was reached. For mesh I, Σ_f^* was found to be 0.0057360 per cm. No attempt was made to find Σ_f^* for mesh II due to time limitations. As such, the test problems outlined below were applied to mesh I. Future work is planned to apply the test problems on mesh II.

The following perturbations were applied:

a) Uniform perturbation of 10 dollar of reactivity per second:

$$\Sigma_f(\bar{r},t) = \Sigma_f^* + \alpha t \ , \qquad \text{in the core}$$

where $\alpha = 0.005893$/cm-sec

b) Local perturbation at the core center of 100 dollar of reactivity per second:

$$\Sigma_f(\bar{r},t) = \Sigma_f^* + \alpha t \delta(\bar{r}_o) \ , \quad \text{in the core}$$

where $\bar{r}_o$ is (0,0,0) and $\alpha = 0.015407/\text{cm-sec}$

c) Local, off-center perturbation:

$$\Sigma_f(\bar{r},t) = \Sigma_f^* + \alpha_i t\delta(\bar{r}_1)\ , \quad i = 1,2,3, \text{ in the core}$$

where

$$\bar{r}_1 = (0,60,40)$$

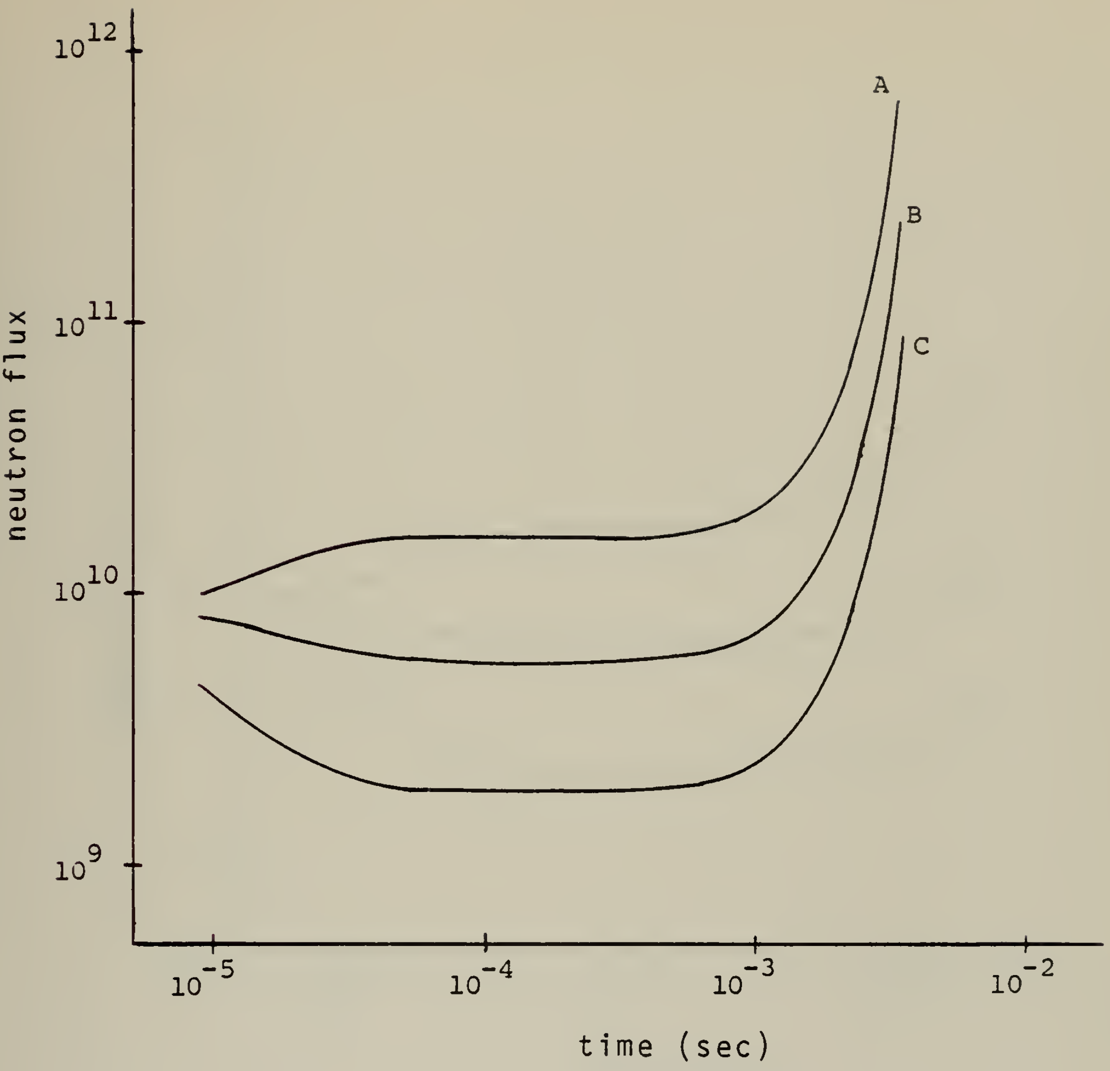
$$\alpha_1 = 0.015407/\text{cm-sec} = 100 \text{ dollar per second}$$

$$\alpha_2 = 0.008123/\text{cm-sec} = 50 \text{ dollar per second}$$

$$\alpha_3 = 0.005894/\text{cm-sec} = 10 \text{ dollar per second}$$

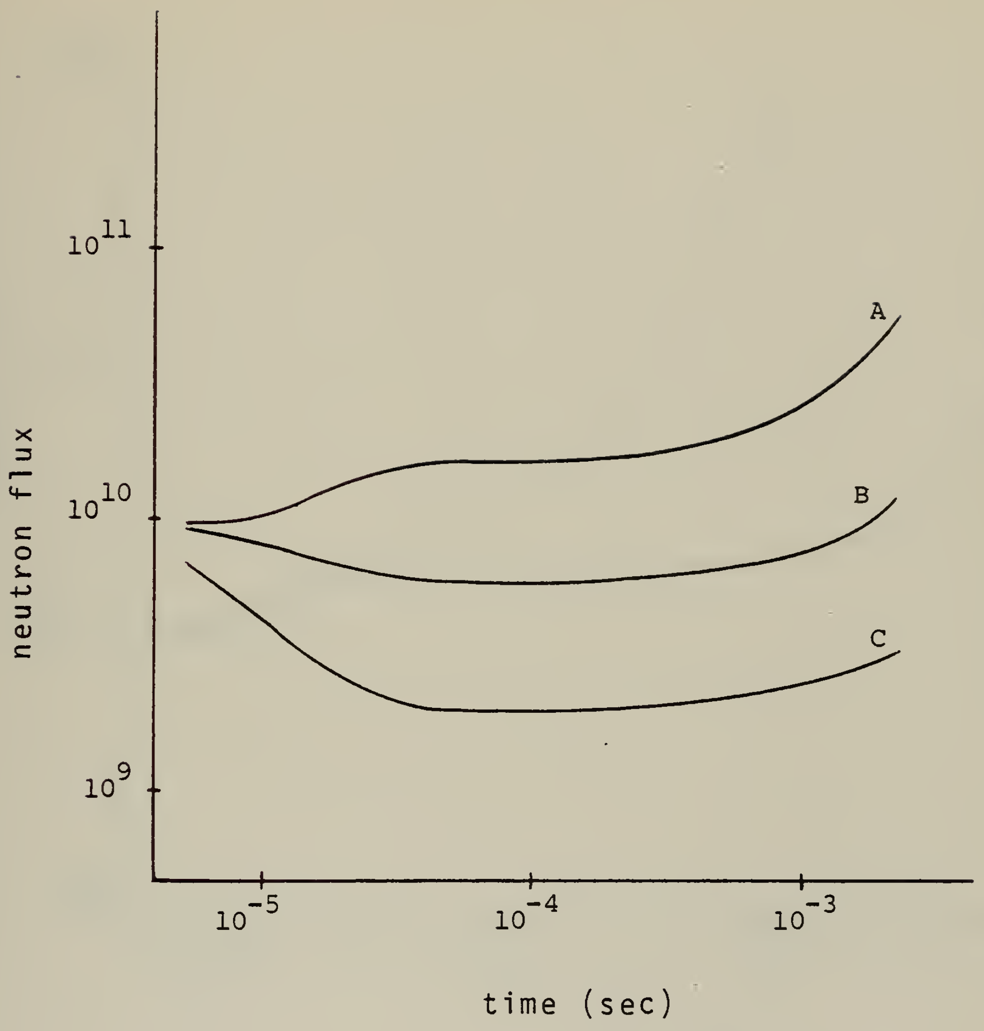
Three test points, (0,0,0), (60,0,0), and (-60,0,80), were selected to trace the neutron time history. For cases a) and b), the neutron flux was plotted at each test point during transience. This is shown in figures 13 and 14. Case c) involved three ramp inputs and were conducted for both the linear and nonlinear reactor equations. The linear and nonlinear responses were compared at each test point for each ramp input and are illustrated in figures 15 through 23. The radial and axial flux distributions at time t = 0.0123 second were plotted for the steady state and the 100 dollar perturbation case. These are embodied in figures 24 and 25. Finally, a neutron flux early time history between mesh I and mesh II was plotted to check the effect of using a finer element mesh. The result is portrayed in figure 26.

Figures 13 and 14 revealed a clear space dependence of the neutron flux during transience as expected. Figures 15 through 23 demonstrated the effects of temperature feedback



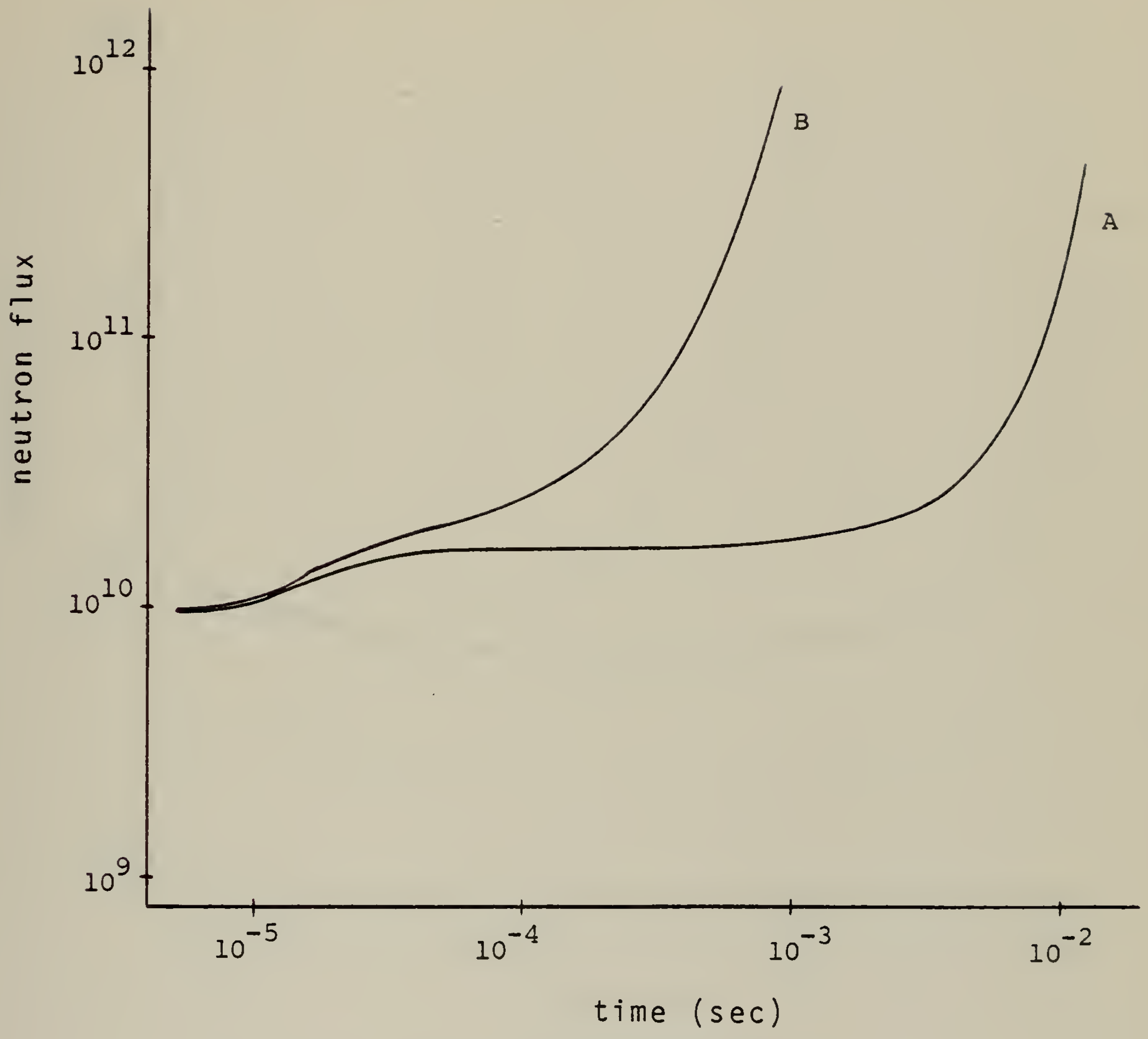
A - neutron flux at test point (0,0,0)
B - neutron flux at test point (60,0,0)
C - neutron flux at test point (-60,0,80)

Figure 13. Neutron flux transient history at three test points with a uniform perturbation of 10 dollar of reactivity per second.



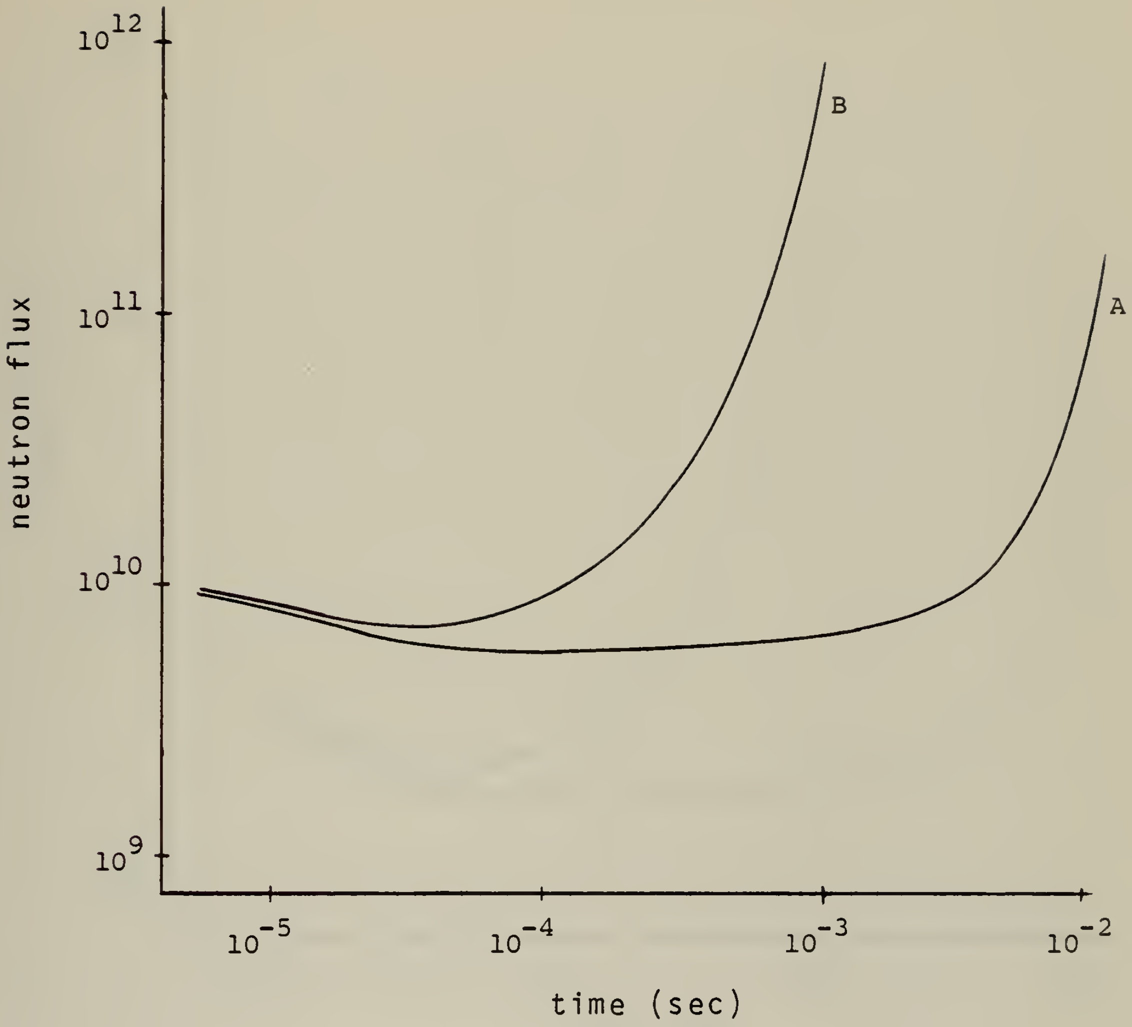
A - neutron flux at test point (0,0,0)
B - neutron flux at test point (60,0,0)
C - neutron flux at test point (-60,0,80)

Figure 14. Neutron flux transient history at various test points for a local central perturbation of 100 dollar/sec of reactivity.



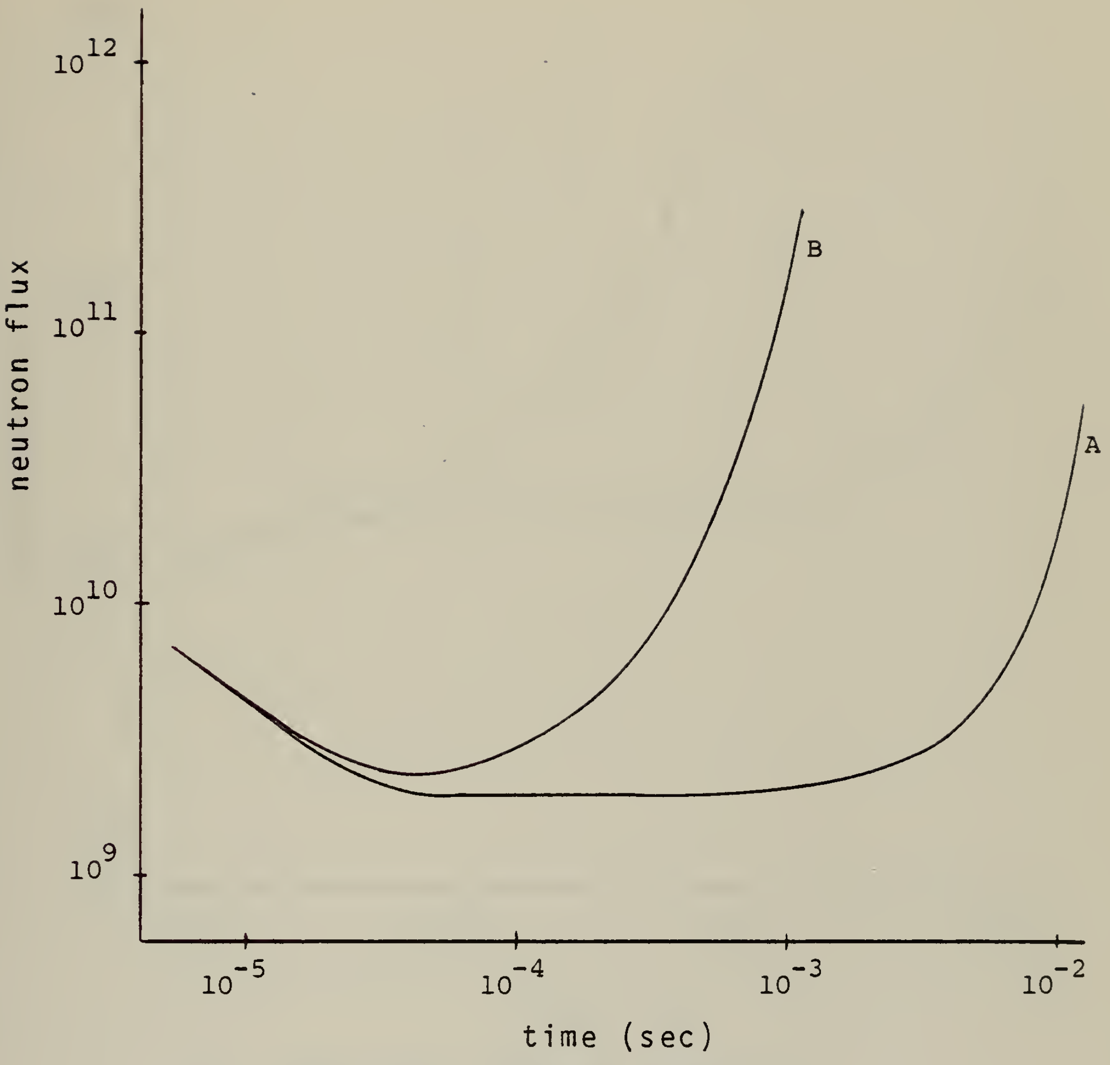
A - nonlinear neutron response
B - linear neutron response

Figure 15. Linear and nonlinear fluxes at (0,0,0) due to a local perturbation of 100 dollar/sec of reactivity at (0,60,40).



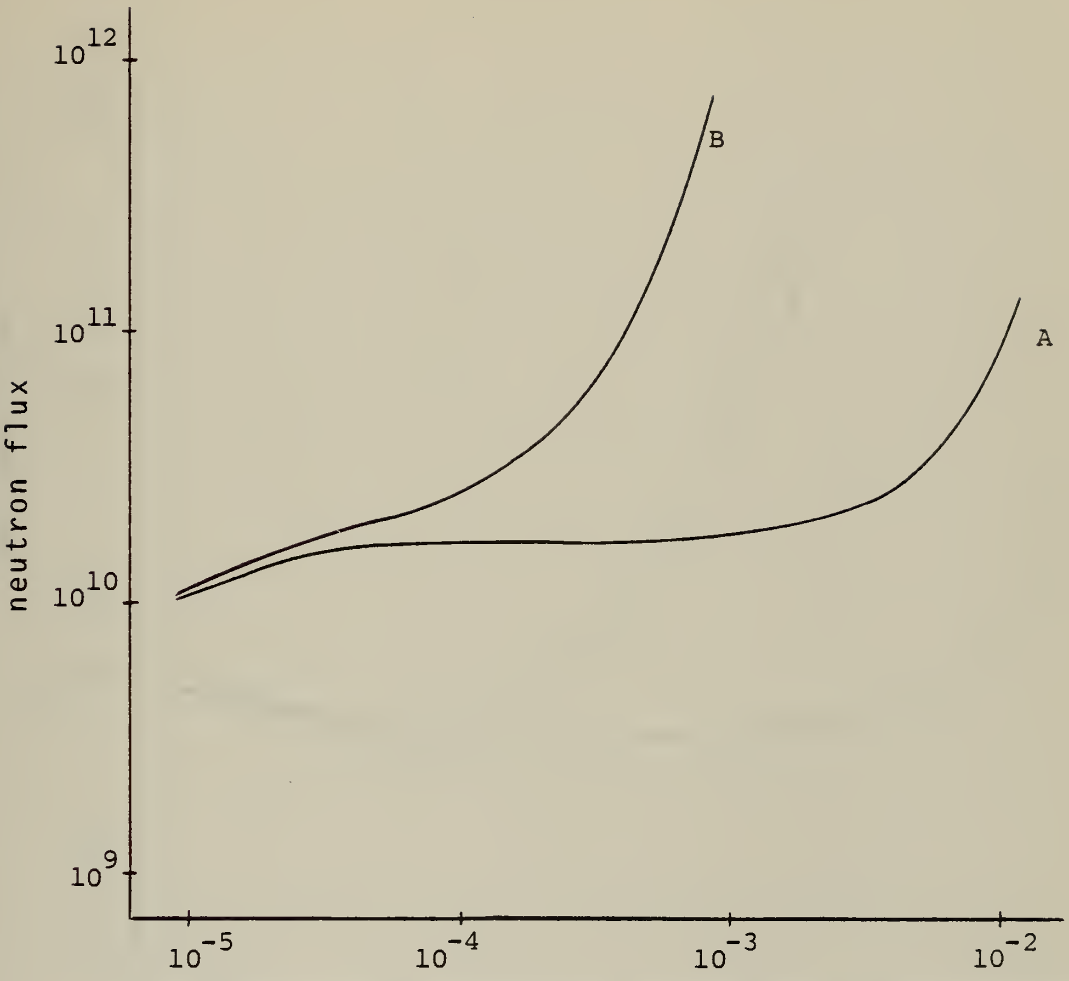
A - nonlinear neutron response
B - linear neutron response

Figure 16. Linear and nonlinear fluxes at (60,0,0) due to a local perturbation of 100 dollar/sec at (0,60,40).



A - nonlinear neutron response
B - linear neutron response

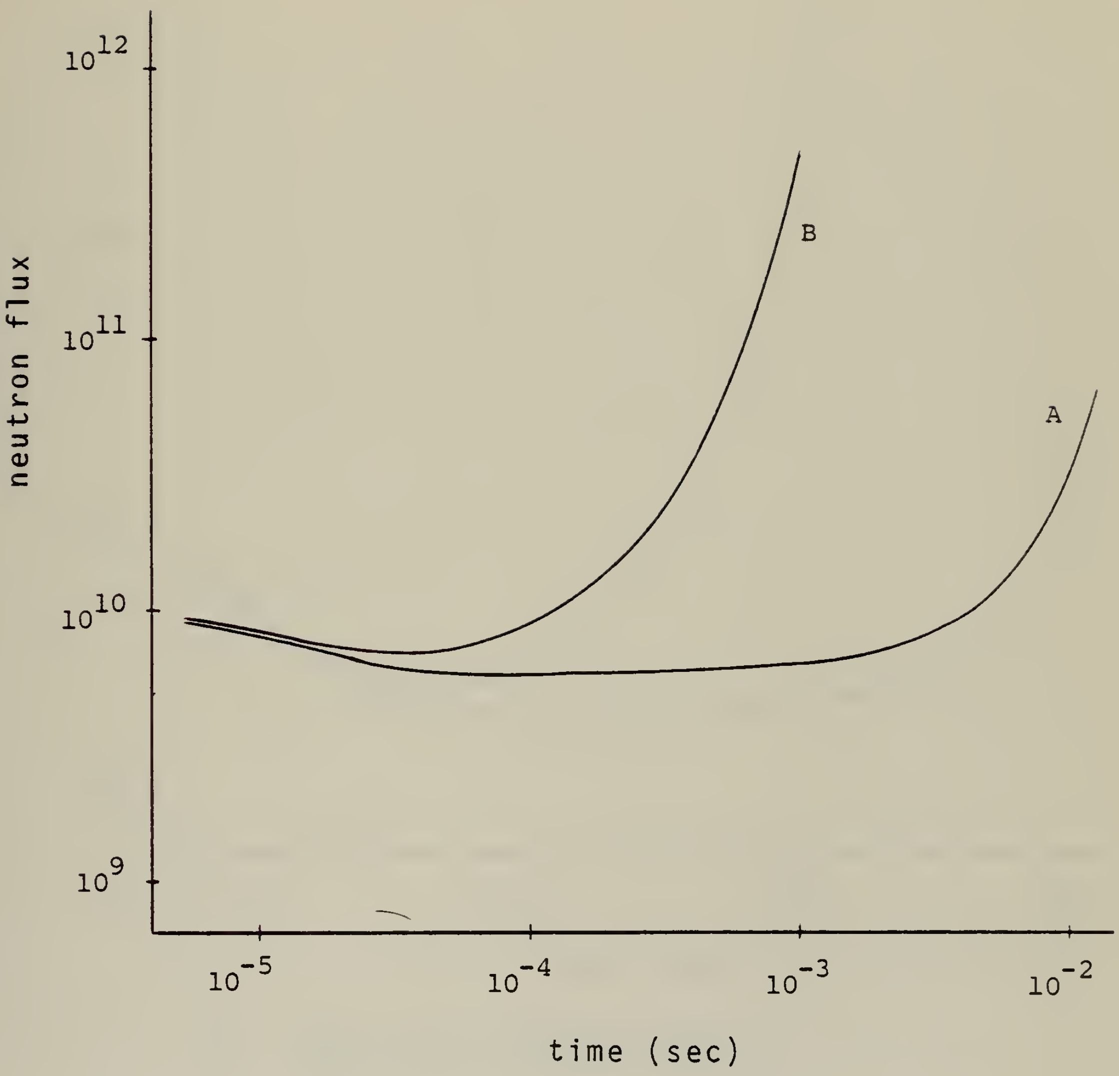
Figure 17. Linear and nonlinear fluxes at (-60,0,80) due to a local perturbation of 100 dollar/sec at (0,60,40).



time (sec)

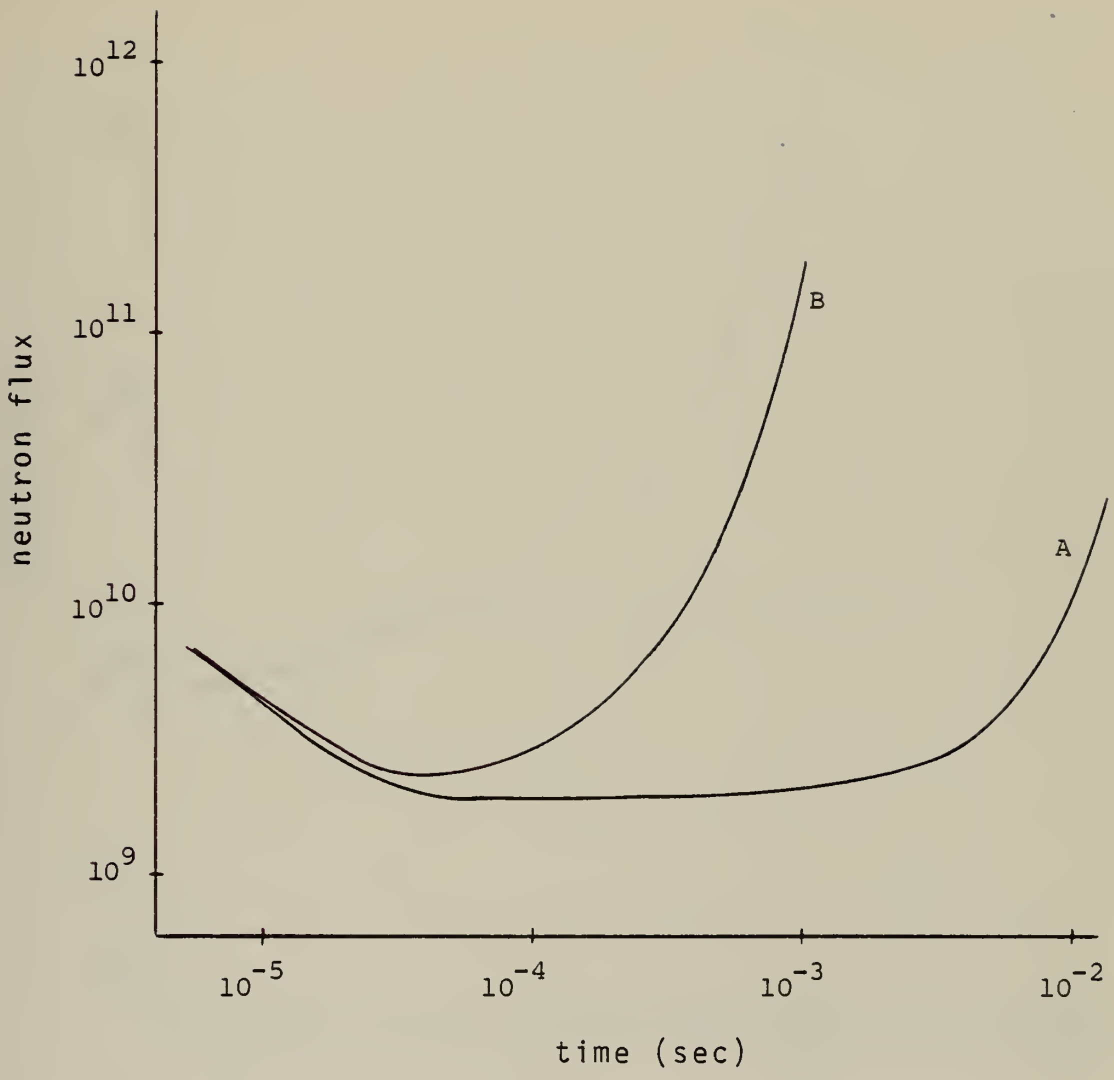
A - nonlinear neutron response
B - linear neutron response

Figure 18. Linear and nonlinear fluxes at (0,0,0) due to a 50 dollar/sec local perturbation at (0,60,40).



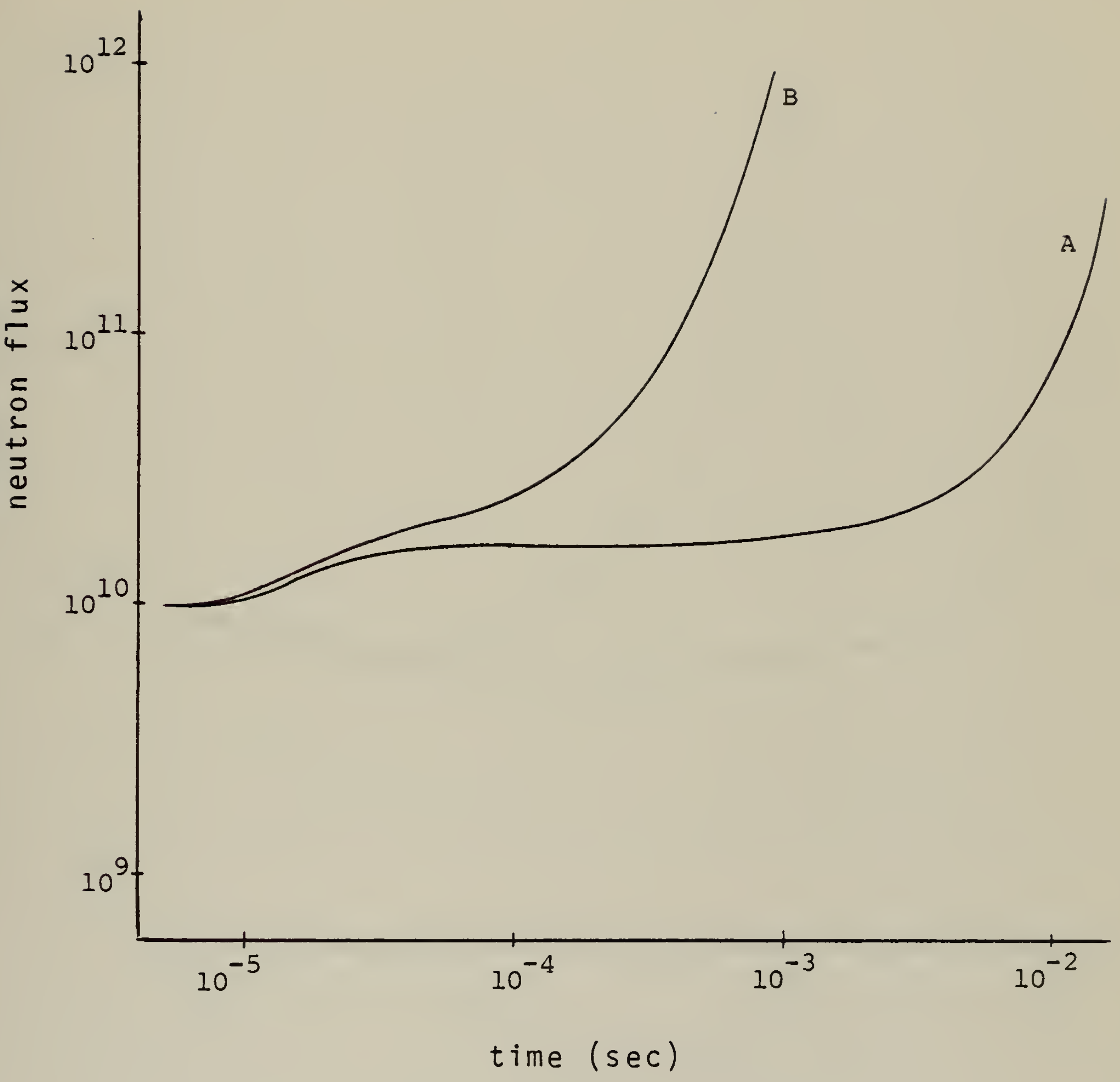
A - nonlinear neutron response
B - linear neutron response

Figure 19. Linear and nonlinear fluxes at (60,0,0) due to a 50 dollar/sec local perturbation at (0,60,40).



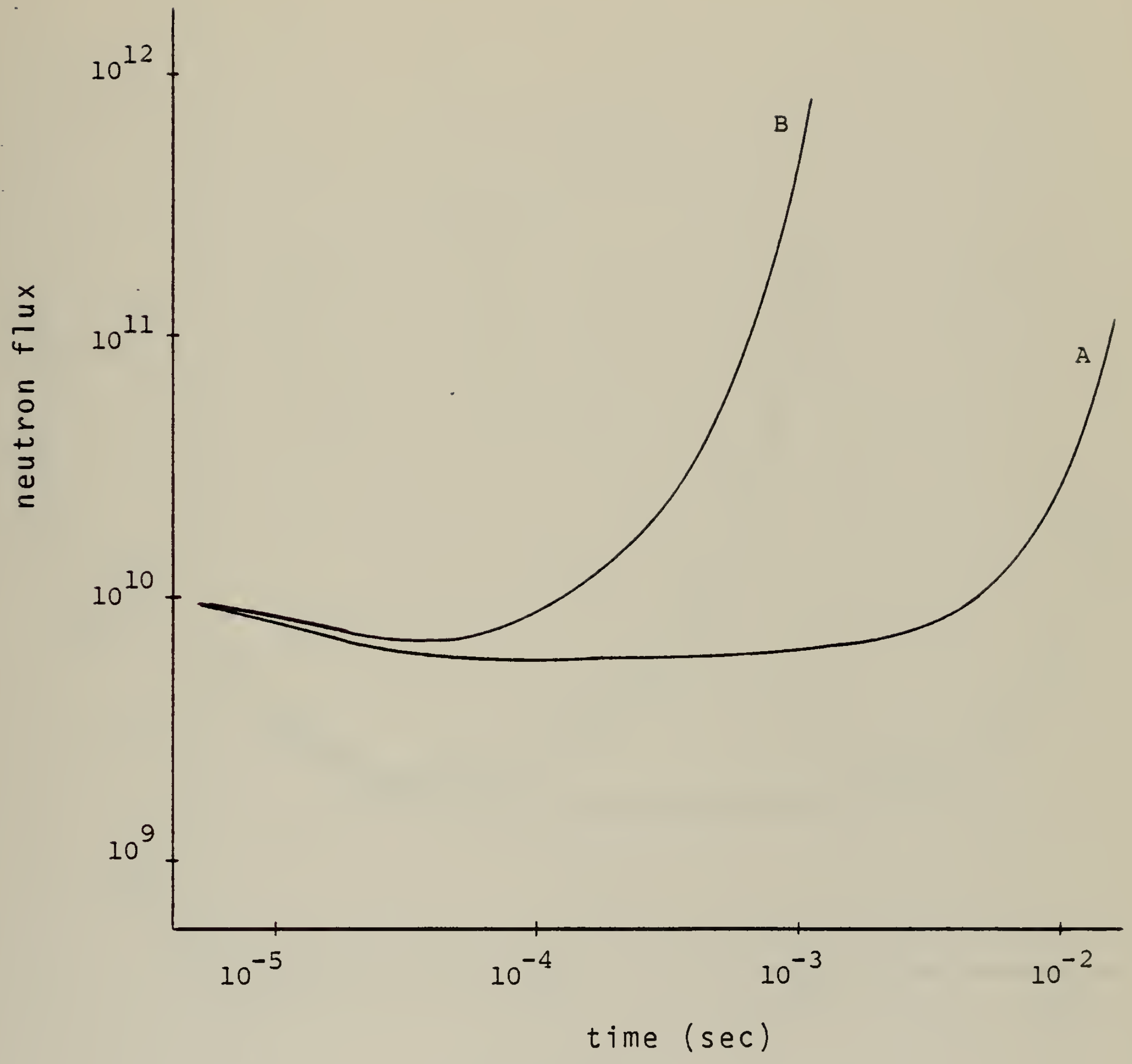
A - nonlinear neutron response
B - linear neutron response

Figure 20. Linear and nonlinear fluxes at (-60,0,80) due to a 50 dollar/sec local perturbation at (0,60,40).



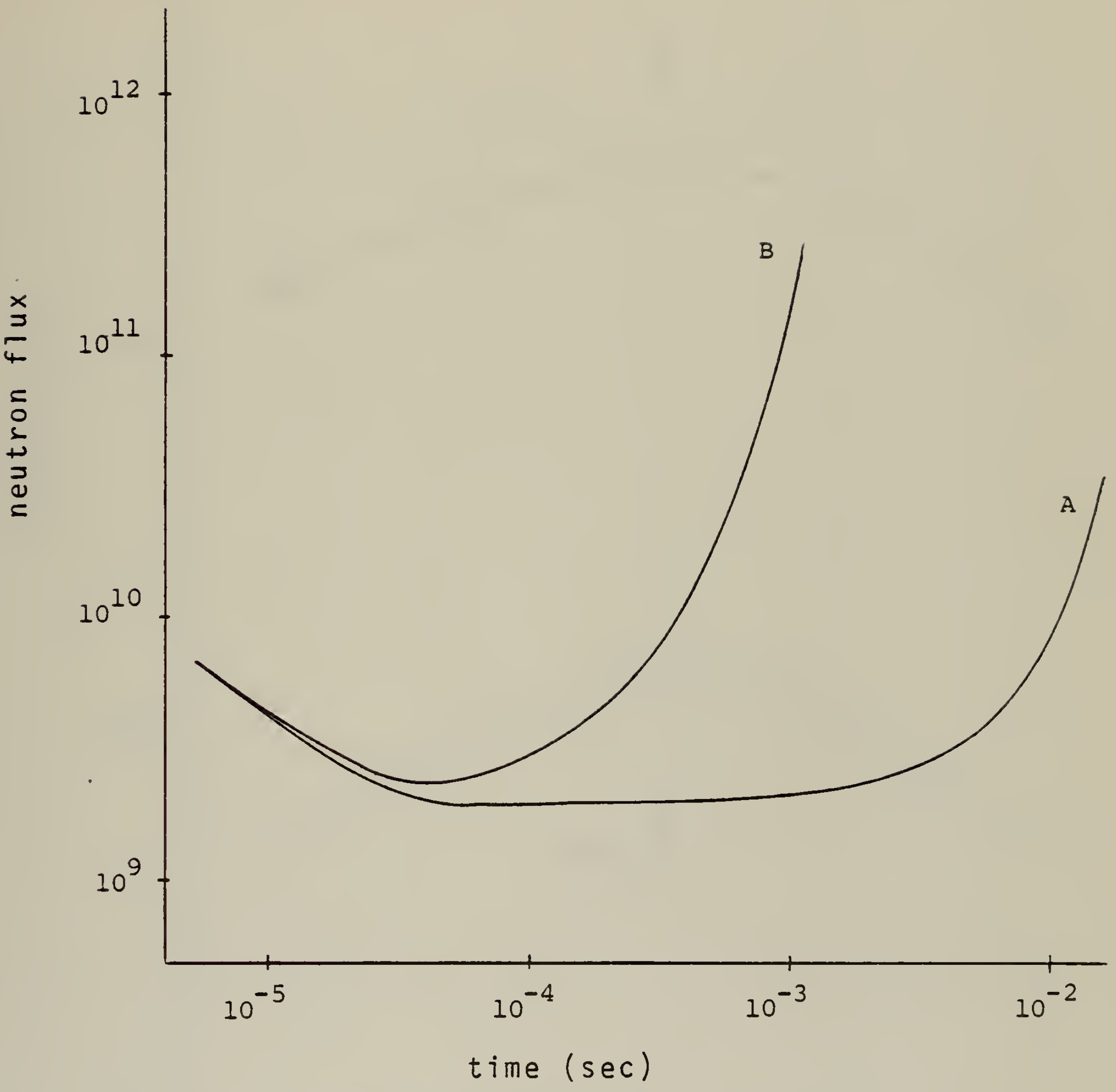
A - nonlinear neutron response
B - linear neutron response

Figure 21. Linear and nonlinear fluxes at (0,0,0) due to a 10 dollar/sec local perturbation at (0,60,40).



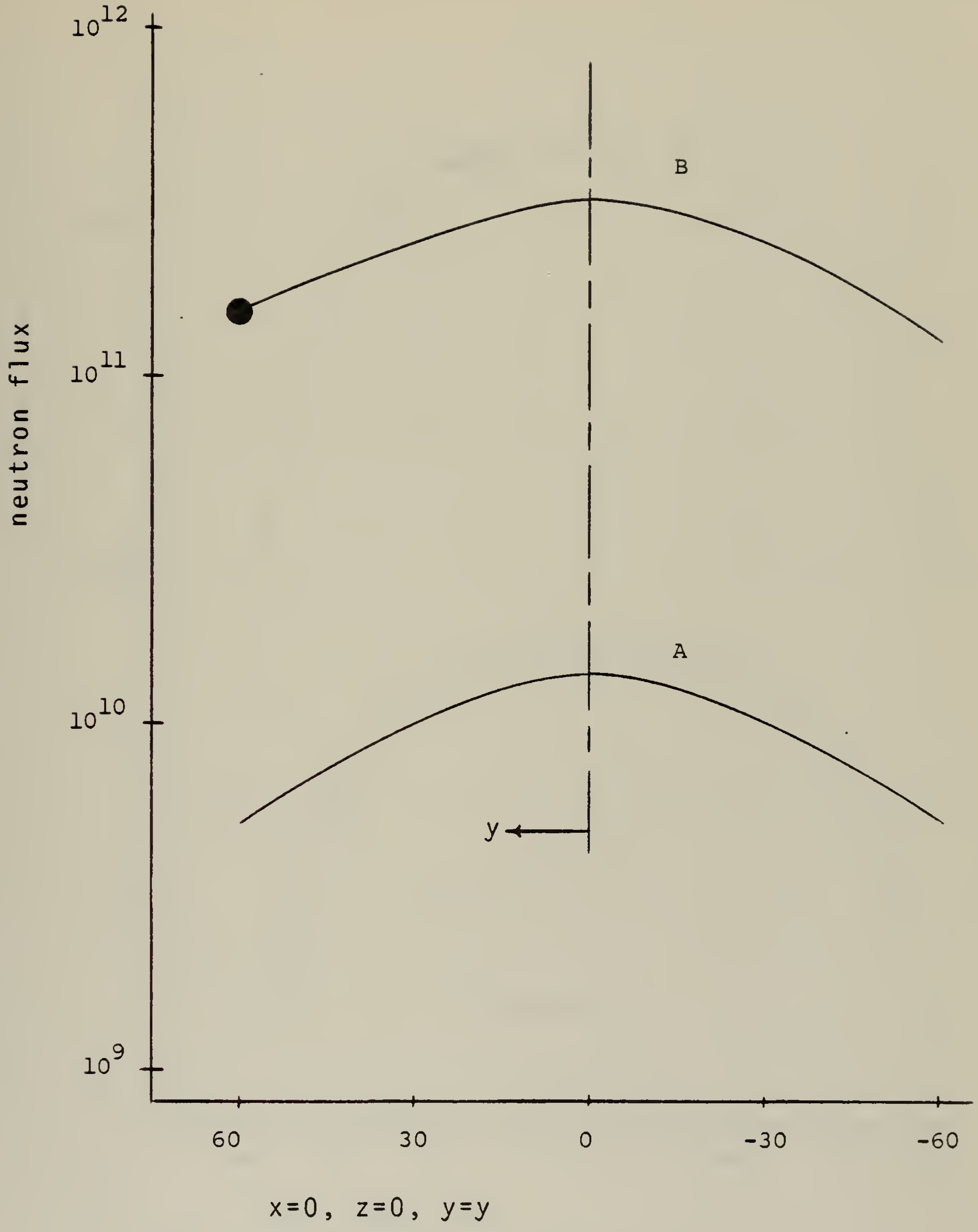
A - nonlinear neutron response
B - linear neutron response

Figure 22. Linear and nonlinear fluxes at (60,0,0) due to a 10 dollar/sec local perturbation at (0,60,40).



A - nonlinear neutron response
B - linear neutron response

Figure 23. Linear and nonlinear fluxes at (-60,0,80) due to a 10 dollar/sec local perturbation at (0,60,40).



● - point of local perturbation

A - neutron flux at steady-state

B - neutron flux under local perturbation

Figure 24. Radial flux distribution for the steady state and 100 dollar/sec local perturbation.

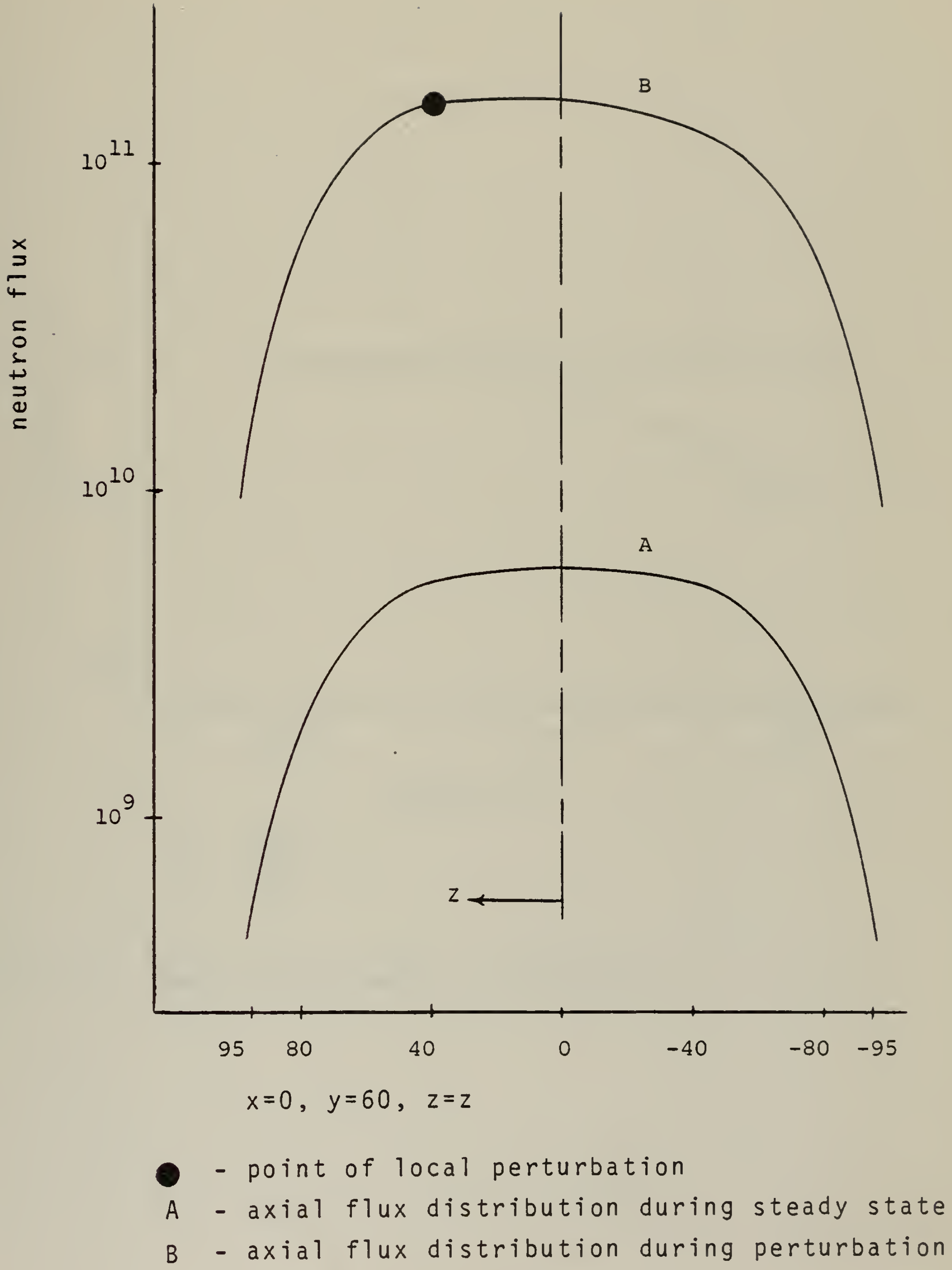


Figure 25. Axial flux distribution for the steady state and 100 dollar/sec local perturbation.

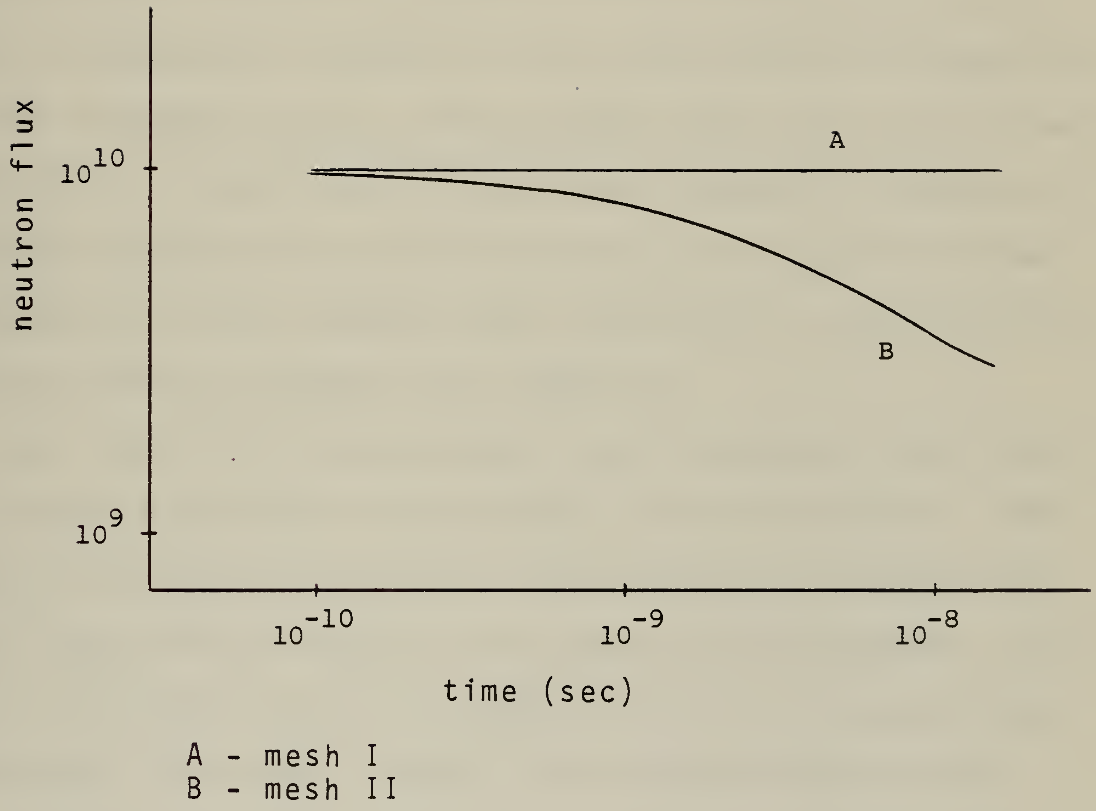


Figure 26. Early time history of the neutron flux at core center using mesh I and mesh II.

and delayed neutrons on the flux. To obtain an idea of the difference in magnitude between the linear and nonlinear flux for the 100 dollar local perturbation, at time t = 10^{-3} second, the linear case predicted a neutron flux at the core center of 2.024×10^{12} neutrons per cm^2 per sec. The nonlinear reactor predicted 1.85×10^{10} neutrons per cm^2 per sec. Although the numbers are not sufficiently accurate due to the crudeness of the finite element mesh, the significant difference in the order of magnitude between the linear and nonlinear neutron flux leads to the belief that the three-dimensional finite element model utilized is predicting the correct trend of neutron flux behavior.

The radial flux distribution for the steady state shown in figure 24 portrays the expected flux distribution. The radial flux distributions for the local perturbation case show a skew distribution at the point of perturbation. The axial flux distribution at steady state is very much symmetric about the center. Again, the expected skew at the point of perturbation is there, as shown in figure 25. Figure 26 gives an indication that the Σ_f^* for finer meshes is higher. However, curve B of figure 26 should be extended to longer times to verify this hypothesis. Curve B, as plotted in figure 26, used four hours of computer time employing the H-compiler of the IBM 360/67.

VI. CONCLUSIONS AND RECOMMENDATIONS

The finite element mesh employed here is crude, and, thus, results that were obtained should be considered as positive indicators rather than numerically conclusive facts. The trend of neutron flux behavior is the major thrust of this work. From the results obtained, it is concluded that the expected patterns of neutron behavior as predicted by the three-dimensional finite element model used do occur. These patterns were best demonstrated by the differences between the linear and nonlinear flux responses and by the spatial flux distribution at steady state and during local perturbation. The three-dimensional quadratic finite element model utilized should produce better results by resorting to a finer mesh. The draw-back to a finer mesh, of course, is the significant increase in computer time and storage requirements. Once accurate results are obtained through finer element meshes, comparisons between three- and two-dimensional models can be attempted.

A mesh generator was not developed for this problem. It is recommended that this be done to ease the transition from one mesh to another and to minimize human error. In addition, a similar calculation using a three-dimensional linear element should be performed to corroborate the results obtained here. It should be particularly noted that this type of problem is highly sensitive to the fission cross-section.

In the search for Σ_f^* , it was necessary to adjust Σ_f to the sixth decimal place. There is no exact method of deriving Σ_f^* due to the highly nonlinear aspect of the problem; therefore, trial and error must be used. Finally, the Gaussian quadrature used to determine the element matrices should be investigated. The cause of the differences in values obtained by using different number of integration points should be established. Was it due to the integration points, the coordinate transformation, or the element shape functions? This is an important question upon which future numerical results could be based.

APPENDIX A

MESH I CONNECTIVITY AND COORDINATES

The connectivity matrix for the 128-element mesh is as follows:

ELEMENT NUMBER	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	3	12	11	10	2	26	28	27	35	37	46	45	44	36
2	1	4	14	13	12	3	26	29	28	35	38	48	47	46	37
3	1	5	16	15	14	4	26	30	29	35	39	50	49	48	38
4	1	6	18	17	16	5	26	31	30	35	40	52	51	50	39
5	1	7	20	19	18	6	26	32	31	35	41	54	53	52	40
6	1	8	22	21	20	7	26	33	32	35	42	56	55	54	41
7	1	9	24	23	22	8	26	34	33	35	43	58	57	56	42
8	1	2	10	25	24	9	26	27	34	35	36	44	59	58	43
9	35	37	46	45	44	36	60	62	61	69	71	80	79	78	70
10	35	38	48	47	46	37	60	63	62	69	72	82	81	80	71
11	35	39	50	49	48	38	60	64	63	69	73	84	83	82	72
12	35	40	52	51	50	39	60	65	64	69	74	86	85	84	73
13	35	41	54	53	52	40	60	66	65	69	75	88	87	86	74
14	35	42	56	55	54	41	60	67	66	69	76	90	89	88	75
15	35	43	58	57	56	42	60	68	67	69	77	92	91	90	76
16	35	36	44	59	58	43	60	61	68	69	70	78	93	92	77
17	69	71	80	79	78	70	118	120	119	425	427	436	435	434	426
18	69	72	82	81	80	71	118	121	120	425	428	438	437	436	427
19	69	73	84	83	82	72	118	122	121	425	429	440	439	438	428
20	69	74	86	85	84	73	118	123	122	425	430	442	441	440	429
21	69	75	88	87	86	74	118	124	123	425	431	444	443	442	430
22	69	76	90	89	88	75	118	125	124	425	432	446	445	444	431
23	69	77	92	91	90	76	118	126	125	425	433	448	447	446	432
24	69	70	78	93	92	77	118	119	126	425	426	434	449	448	433
25	78	95	379	378	377	94	119	410	409	434	451	476	475	474	450
26	379	95	78	79	80	96	410	119	120	476	451	434	435	436	452
27	80	97	381	380	379	96	120	411	410	436	453	478	477	476	452
28	80	98	383	382	381	97	120	412	411	436	454	480	479	478	453
29	383	98	80	81	82	99	412	120	121	480	454	436	437	438	455
30	82	100	385	384	383	99	121	413	412	438	456	482	481	480	455
31	82	101	387	386	385	100	121	414	413	438	457	484	483	482	456
32	387	101	82	83	84	102	414	121	122	484	457	438	439	440	458
33	84	103	389	388	387	102	122	415	414	440	459	486	485	484	458
34	84	104	391	390	389	103	122	416	415	440	460	488	487	486	459
35	391	104	84	85	86	105	416	122	123	488	460	440	441	442	461
36	86	106	393	392	391	105	123	417	416	442	462	490	489	488	461
37	86	107	395	394	393	106	123	418	417	442	463	492	491	490	462

```
38   395   107    86    87    88   108   418   123   124   492   463   442   443   444   464
39    88   109   397   396   395   108   124   419   418   444   465   494   493   492   464
40    88   110   399   398   397   109   124   420   419   444   466   496   495   494   465
41   399   110    88    89    90   111   420   124   125   496   466   444   445   446   467
42    90   112   401   400   399   111   125   421   420   446   468   498   497   496   467
43    90   113   403   402   401   112   125   422   421   446   469   500   499   498   468
44   403   113    90    91    92   114   422   125   126   500   469   446   447   448   470
45    92   115   405   404   403   114   126   423   422   448   471   502   501   500   470
46    92   116   407   406   405   115   126   424   423   448   472   504   503   502   471
47   407   116    92    93    78   117   424   126   119   504   472   448   449   434   473
48    78    94   377   408   407   117   119   409   424   434   450   474   505   504   473
49    44   128   331   330   329   127    61   362   361    78    95   379   378   377    94
50   331   128    44    45    46   129   362    61    62   379    95    78    79    80    96
51    46   130   333   332   331   129    62   363   362    80    97   381   380   379    96
52    46   131   335   334   333   130    62   364   363    80    98   383   382   381    97
53   335   131    46    47    48   132   364    62    63   383    98    80    81    82    99
54    48   133   337   336   335   132    63   365   364    82   100   385   384   383    99
55    48   134   339   338   337   133    63   366   365    82   101   387   386   385   100
56   339   134    48    49    50   135   366    63    64   387   101    82    83    84   102
57    50   136   341   340   339   135    64   367   366    84   103   389   388   387   102
58    50   137   343   342   341   136    64   368   367    84   104   391   390   389   103
59   343   137    50    51    52   138   368    64    65   391   104    84    85    86   105
60    52   139   345   344   343   138    65   369   368    86   106   393   392   391   105
61    52   140   347   346   345   139    65   370   369    86   107   395   394   393   106
62   347   140    52    53    54   141   370    65    66   395   107    86    87    88   108
63    54   142   349   348   347   141    66   371   370    88   109   397   396   395   108
64    54   143   351   350   349   142    66   372   371    88   110   399   398   397   109
65   351   143    54    55    56   144   372    66    67   399   110    88    89    90   111
66    56   145   353   352   351   144    67   373   372    90   112   401   400   399   111
67    56   146   355   354   353   145    67   374   373    90   113   403   402   401   112
68   355   146    56    57    58   147   374    67    68   403   113    90    91    92   114
69    58   148   357   356   355   147    68   375   374    92   115   405   404   403   114
70    58   149   359   358   357   148    68   376   375    92   116   407   406   405   115
71   359   149    58    59    44   150   376    68    61   407   116    92    93    78   117
72    44   127   329   360   359   150    61   361   376    78    94   377   408   407   117
73    10   152   283   282   281   151    27   314   313    44   128   331   330   329   127
74   283   152    10    11    12   153   314    27    28   331   128    44    45    46   129
75    12   154   285   284   283   153    28   315   314    46   130   333   332   331   129
76    12   155   287   286   285   154    28   316   315    46   131   335   334   333   130
77   287   155    12    13    14   156   316    28    29   335   131    46    47    48   132
78    14   157   289   288   287   156    29   317   316    48   133   337   336   335   132
79    14   158   291   290   289   157    29   318   317    48   134   339   338   337   133
80   291   158    14    15    16   159   318    29    30   339   134    48    49    50   135
81    16   160   293   292   291   159    30   319   318    50   136   341   340   339   135
82    16   161   295   294   293   160    30   320   319    50   137   343   342   341   136
83   295   161    16    17    18   162   320    30    31   343   137    50    51    52   138
84    18   163   297   296   295   162    31   321   320    52   139   345   344   343   138
85    18   164   299   298   297   163    31   322   321    52   140   347   346   345   139
```

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 86 299 164  18  19  20 165 322  31  32 347 140  52  53  54 141
 87  20 166 301 300 299 165  32 323 322  54 142 349 348 347 141
 88  20 167 303 302 301 166  32 324 323  54 143 351 350 349 142
 89 303 167  20  21  22 168 324  32  33 351 143  54  55  56 144
 90  22 169 305 304 303 168  33 325 324  56 145 353 352 351 144
 91  22 170 307 306 305 169  33 326 325  56 146 355 354 353 145
 92 307 170  22  23  24 171 326  33  34 355 146  56  57  58 147
 93  24 172 309 308 307 171  34 327 326  58 148 357 356 355 147
 94  24 173 311 310 309 172  34 328 327  58 149 359 358 357 148
 95 311 173  24  25  10 174 328  34  27 359 149  58  59  44 150
 96  10 151 281 312 311 174  27 313 328  44 127 329 360 359 150
 97 184 186 195 194 193 185 175 177 176   1   3  12  11  10   2
 98 184 187 197 196 195 186 175 178 177   1   4  14  13  12   3
 99 184 188 199 198 197 187 175 179 178   1   5  16  15  14   4
100 184 189 201 200 199 188 175 180 179   1   6  18  17  16   5
101 184 190 203 202 201 189 175 181 180   1   7  20  19  18   6
102 184 191 205 204 203 190 175 182 181   1   8  22  21  20   7
103 184 192 207 206 205 191 175 183 182   1   9  24  23  22   8
104 184 185 193 208 207 192 175 176 183   1   2  10  25  24   9
105 193 210 235 234 233 209 176 266 265  10 152 283 282 281 151
106 235 210 193 194 195 211 266 175 177 283 152  10  11  12 153
107 195 212 237 236 235 211 177 267 266  12 154 285 284 283 153
108 195 213 239 238 237 212 177 268 267  12 155 287 286 285 154
109 239 213 195 196 197 214 268 177 178 287 155  12  13  14 156
110 197 215 241 240 239 214 178 269 268  14 157 289 288 287 156
111 197 216 243 242 241 215 178 270 269  14 158 291 290 289 157
112 243 216 197 198 199 217 270 178 179 291 158  14  15  16 159
113 199 218 245 244 243 217 179 271 270  16 160 293 292 291 159
114 199 219 247 246 245 218 179 272 271  16 161 295 294 293 160
115 247 219 199 200 201 220 272 179 180 295 161  16  17  18 162
116 201 221 249 248 247 220 180 273 272  18 163 297 296 295 162
117 201 222 251 250 249 221 180 274 273  18 164 299 298 297 163
118 251 222 201 202 203 223 274 180 181 299 164  18  19  20 165
119 203 224 253 252 251 223 181 275 274  20 166 301 300 299 165
120 203 225 255 254 253 224 181 276 275  20 167 303 302 301 166
121 255 225 203 204 205 226 276 181 182 303 167  20  21  22 168
122 205 227 257 256 255 226 182 277 276  22 169 305 304 303 168
123 205 228 259 258 257 227 182 278 277  22 170 307 306 305 169
124 259 228 205 206 207 229 278 182 183 307 170  22  23  24 171
125 207 230 261 260 259 229 183 279 278  24 172 309 308 307 171
126 207 231 263 262 261 230 183 280 279  24 173 311 310 309 172
127 263 231 207 208 193 232 280 183 176 311 173  24  25  10 174
128 193 209 233 264 263 232 176 265 280  10 151 281 312 311 174
```

The coordinates of the 128-element mesh are as follows:

NODE NUMBER	X	Y	Z
1	0.0	0.0	80.00
2	0.0	30.00	80.00
3	21.21	21.21	80.00
4	30.00	0.0	80.00
5	21.21	-21.21	80.00
6	0.0	-30.00	80.00
7	-21.21	-21.21	80.00
8	-30.00	0.0	80.00
9	-21.21	21.21	80.00
10	0.0	60.00	80.00
11	22.96	55.43	80.00
12	42.43	42.43	80.00
13	55.43	22.96	80.00
14	60.00	0.0	80.00
15	55.43	-22.96	80.00
16	42.43	-42.43	80.00
17	22.96	-55.43	80.00
18	0.0	-60.00	80.00
19	-22.96	-55.43	80.00
20	-42.43	-42.43	80.00
21	-55.43	-22.96	80.00
22	-60.00	0.0	80.00
23	-55.43	22.96	80.00
24	-42.43	42.43	80.00
25	-22.96	55.43	80.00
26	0.0	0.0	40.00
27	0.0	60.00	40.00
28	42.43	42.43	40.00
29	60.00	0.0	40.00
30	42.43	-42.43	40.00
31	0.0	-60.00	40.00
32	-42.43	-42.43	40.00
33	-60.00	0.0	40.00
34	-42.43	42.43	40.00
35	0.0	0.0	0.0
36	0.0	30.00	0.0
37	21.21	21.21	0.0
38	30.00	0.0	0.0
39	21.21	-21.21	0.0
40	0.0	-30.00	0.0
41	-21.21	-21.21	0.0
42	-30.00	0.0	0.0
43	-21.21	21.21	0.0
44	0.0	60.00	0.0

45	22.96	55.43	0.0
46	42.43	42.43	0.0
47	55.43	22.96	0.0
48	60.00	0.0	0.0
49	55.43	-22.96	0.0
50	42.43	-42.43	0.0
51	22.96	-55.43	0.0
52	0.0	-60.00	0.0
53	-22.96	-55.43	0.0
54	-42.43	-42.43	0.0
55	-55.43	-22.96	0.0
56	-60.00	0.0	0.0
57	-55.43	22.96	0.0
58	-42.43	42.43	0.0
59	-22.96	55.43	0.0
60	0.0	0.0	-40.00
61	0.0	60.00	-40.00
62	42.43	42.43	-40.00
63	60.00	0.0	-40.00
64	42.43	-42.43	-40.00
65	0.0	-60.00	-40.00
66	-42.43	-42.43	-40.00
67	-60.00	0.0	-40.00
68	-42.43	42.43	-40.00
69	0.0	0.0	-80.00
70	0.0	30.00	-80.00
71	21.21	21.21	-80.00
72	30.00	0.0	-80.00
73	21.21	-21.21	-80.00
74	0.0	-30.00	-80.00
75	-21.21	-21.21	-80.00
76	-30.00	0.0	-80.00
77	-21.21	21.21	-80.00
78	0.0	60.00	-80.00
79	22.96	55.43	-80.00
80	42.43	42.43	-80.00
81	55.43	22.96	-80.00
82	60.00	0.0	-80.00
83	55.43	-22.96	-80.00
84	42.43	-42.43	-80.00
85	22.96	-55.43	-80.00
86	0.0	-60.00	-80.00
87	-22.96	-55.43	-80.00
88	-42.43	-42.43	-80.00
89	-55.43	-22.96	-80.00
90	-60.00	0.0	-80.00
91	-55.43	22.96	-80.00
92	-42.43	42.43	-80.00

93	-22.96	55.43	-80.00
94	0.0	75.00	-80.00
95	16.80	71.60	-80.00
96	38.10	62.90	-80.00
97	53.03	53.03	-80.00
98	62.80	38.60	-80.00
99	71.30	17.00	-80.00
100	75.00	0.0	-80.00
101	71.30	-17.00	-80.00
102	62.80	-38.60	-80.00
103	53.03	-53.03	-80.00
104	38.10	-62.90	-80.00
105	17.10	-71.80	-80.00
106	0.0	-75.00	-80.00
107	-16.90	-71.80	-80.00
108	-38.50	-62.80	-80.00
109	-53.03	-53.03	-80.00
110	-62.70	-38.60	-80.00
111	-71.30	-17.10	-80.00
112	-75.00	0.0	-80.00
113	-71.30	17.10	-80.00
114	-62.90	38.30	-80.00
115	-53.03	53.03	-80.00
116	-38.60	62.80	-80.00
117	-17.30	71.30	-80.00
118	0.0	0.0	-95.00
119	0.0	60.00	-95.00
120	42.43	42.43	-95.00
121	60.00	0.0	-95.00
122	42.43	-42.43	-95.00
123	0.0	-60.00	-95.00
124	-42.43	-42.43	-95.00
125	-60.00	0.0	-95.00
126	-42.43	42.43	-95.00
127	0.0	75.00	0.0
128	16.80	71.60	0.0
129	38.10	62.90	0.0
130	53.03	53.03	0.0
131	62.80	38.60	0.0
132	71.30	17.00	0.0
133	75.00	0.0	0.0
134	71.30	-17.00	0.0
135	62.80	-38.60	0.0
136	53.03	-53.03	0.0
137	38.10	-62.90	0.0
138	17.10	-71.80	0.0
139	0.0	-75.00	0.0
140	-16.90	-71.80	0.0

141	-38.50	-62.80	0.0
142	-53.03	-53.03	0.0
143	-62.70	-38.60	0.0
144	-71.30	-17.10	0.0
145	-75.00	0.0	0.0
146	-71.30	17.10	0.0
147	-62.90	38.30	0.0
148	-53.03	53.03	0.0
149	-38.60	62.80	0.0
150	-17.30	71.30	0.0
151	0.0	75.00	80.00
152	16.80	71.60	80.00
153	38.10	62.90	80.00
154	53.03	53.03	80.00
155	62.80	38.60	80.00
156	71.30	17.00	80.00
157	75.00	0.0	80.00
158	71.30	-17.00	80.00
159	62.80	-38.60	80.00
160	53.03	-53.03	80.00
161	38.10	-62.90	80.00
162	17.10	-71.80	80.00
163	0.0	-75.00	80.00
164	-16.90	-71.80	80.00
165	-38.50	-62.80	80.00
166	-53.03	-53.03	80.00
167	-62.70	-38.60	80.00
168	-71.30	-17.10	80.00
169	-75.00	0.0	80.00
170	-71.30	17.10	80.00
171	-62.90	38.30	80.00
172	-53.03	53.03	80.00
173	-38.60	62.80	80.00
174	-17.30	71.30	80.00
175	0.0	0.0	95.00
176	0.0	60.00	95.00
177	42.43	42.43	95.00
178	60.00	0.0	95.00
179	42.43	-42.43	95.00
180	0.0	-60.00	95.00
181	-42.42	-42.43	95.00
182	-60.00	0.0	95.00
183	-42.43	42.43	95.00
184	0.0	0.0	110.00
185	0.0	30.00	110.00
186	21.21	21.21	110.00
187	30.00	0.0	110.00
188	21.21	-21.21	110.00

189	0.0	-30.00	110.00
190	-21.21	-21.21	110.00
191	-30.00	0.0	110.00
192	-21.21	21.21	110.00
193	0.0	60.00	110.00
194	22.96	55.43	110.00
195	42.43	42.43	110.00
196	55.43	22.96	110.00
197	60.00	0.0	110.00
198	55.43	-22.96	110.00
199	42.43	-42.43	110.00
200	22.96	-55.43	110.00
201	0.0	-60.00	110.00
202	-22.96	-55.43	110.00
203	-42.43	-42.43	110.00
204	-55.43	-22.96	110.00
205	-60.00	0.0	110.00
206	-55.43	22.96	110.00
207	-42.43	42.43	110.00
208	-22.96	55.43	110.00
209	0.0	75.00	110.00
210	16.80	71.60	110.00
211	38.10	62.90	110.00
212	53.03	53.03	110.00
213	62.80	38.60	110.00
214	71.30	17.00	110.00
215	75.00	0.0	110.00
216	71.30	-17.00	110.00
217	62.80	-38.60	110.00
218	53.03	-53.03	110.00
219	38.10	-62.90	110.00
220	17.10	-71.80	110.00
221	0.0	-75.00	110.00
222	-16.90	-71.80	110.00
223	-38.50	-62.80	110.00
224	-53.03	-53.03	110.00
225	-62.70	-38.60	110.00
226	-71.30	-17.10	110.00
227	-75.00	0.0	110.00
228	-71.30	17.10	110.00
229	-62.90	38.30	110.00
230	-53.03	53.03	110.00
231	-38.60	62.80	110.00
232	-17.30	71.30	110.00
233	0.0	90.00	110.00
234	17.56	88.27	110.00
235	34.44	83.15	110.00
236	50.00	74.83	110.00

237	63.64	63.64	110.00
238	74.83	50.00	110.00
239	83.15	34.44	110.00
240	88.27	17.56	110.00
241	90.00	0.0	110.00
242	88.27	-17.56	110.00
243	83.15	-34.44	110.00
244	74.83	-50.00	110.00
245	63.64	-63.64	110.00
246	50.00	-74.83	110.00
247	34.44	-83.15	110.00
248	17.56	-88.27	110.00
249	0.0	-90.00	110.00
250	-17.56	-88.27	110.00
251	-34.44	-83.15	110.00
252	-50.00	-74.83	110.00
253	-63.64	-63.64	110.00
254	-74.83	-50.00	110.00
255	-83.15	-34.44	110.00
256	-88.27	-17.56	110.00
257	-90.00	0.0	110.00
258	-88.27	17.56	110.00
259	-83.15	34.44	110.00
260	-74.83	50.00	110.00
261	-63.64	63.64	110.00
262	-50.00	74.83	110.00
263	-34.44	83.15	110.00
264	-17.56	88.27	110.00
265	0.0	90.00	95.00
266	34.44	83.15	95.00
267	63.64	63.64	95.00
268	83.15	34.44	95.00
269	90.00	0.0	95.00
270	83.15	-34.44	95.00
271	63.64	-63.64	95.00
272	34.44	-83.15	95.00
273	0.0	-90.00	95.00
274	-34.44	-83.15	95.00
275	-63.64	-63.64	95.00
276	-83.15	-34.44	95.00
277	-90.00	0.0	95.00
278	-83.15	34.44	95.00
279	-63.64	63.64	95.00
280	-34.44	83.15	95.00
281	0.0	90.00	80.00
282	17.56	88.27	80.00
283	34.44	83.15	80.00
284	50.00	74.83	80.00

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285        63.64        63.64        80.00
286        74.83        50.00        80.00
287        83.15        34.44        80.00
288        88.27        17.56        80.00
289        90.00         0.0         80.00
290        88.27       -17.56        80.00
291        83.15       -34.44        80.00
292        74.83       -50.00        80.00
293        63.64       -63.64        80.00
294        50.00       -74.83        80.00
295        34.44       -83.15        80.00
296        17.56       -88.27        80.00
297         0.0        -90.00        80.00
298       -17.56       -88.27        80.00
299       -34.44       -83.15        80.00
300       -50.00       -74.83        80.00
301       -63.64       -63.64        80.00
302       -74.83       -50.00        80.00
303       -83.15       -34.44        80.00
304       -88.27       -17.56        80.00
305       -90.00         0.0         80.00
306       -88.27        17.56        80.00
307       -83.15        34.44        80.00
308       -74.83        50.00        80.00
309       -63.64        63.64        80.00
310       -50.00        74.83        80.00
311       -34.44        83.15        80.00
312       -17.56        88.27        80.00
313         0.0         90.00        40.00
314        34.44        83.15        40.00
315        63.64        63.64        40.00
316        83.15        34.44        40.00
317        90.00         0.0         40.00
318        83.15       -34.44        40.00
319        63.64       -63.64        40.00
320        34.44       -83.15        40.00
321         0.0        -90.00        40.00
322       -34.44       -83.15        40.00
323       -63.64       -63.64        40.00
324       -83.15       -34.44        40.00
325       -90.00         0.0         40.00
326       -83.15        34.44        40.00
327       -63.64        63.64        40.00
328       -34.44        83.15        40.00
329         0.0         90.00         0.0
330        17.56        88.27         0.0
331        34.44        83.15         0.0
332        50.00        74.83         0.0
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333      63.64      63.64      0.0
334      74.83      50.00      0.0
335      83.15      34.44      0.0
336      88.27      17.56      0.0
337      90.00       0.0       0.0
338      88.27     -17.56      0.0
339      83.15     -34.44      0.0
340      74.83     -50.00      0.0
341      63.64     -63.64      0.0
342      50.00     -74.83      0.0
343      34.44     -83.15      0.0
344      17.56     -88.27      0.0
345       0.0      -90.00      0.0
346     -17.56     -88.27      0.0
347     -34.44     -83.15      0.0
348     -50.00     -74.83      0.0
349     -63.64     -63.64      0.0
350     -74.83     -50.00      0.0
351     -83.15     -34.44      0.0
352     -88.27     -17.56      0.0
353     -90.00       0.0       0.0
354     -88.27      17.56      0.0
355     -83.15      34.44      0.0
356     -74.83      50.00      0.0
357     -63.64      63.64      0.0
358     -50.00      74.83      0.0
359     -34.44      83.15      0.0
360     -17.56      88.27      0.0
361       0.0       90.00    -40.00
362      34.44      83.15    -40.00
363      63.64      63.64    -40.00
364      83.15      34.44    -40.00
365      90.00       0.0     -40.00
366      83.15     -34.44    -40.00
367      63.64     -63.64    -40.00
368      34.44     -83.15    -40.00
369       0.0      -90.00    -40.00
370     -34.44     -83.15    -40.00
371     -63.64     -63.64    -40.00
372     -83.15     -34.44    -40.00
373     -90.00       0.0     -40.00
374     -83.15      34.44    -40.00
375     -63.64      63.64    -40.00
376     -34.44      83.15    -40.00
377       0.0       90.00    -80.00
378      17.56      88.27    -80.00
379      34.44      83.15    -80.00
380      50.00      74.83    -80.00
```

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381      63.64      63.64     -80.00
382      74.83      50.00     -80.00
383      83.15      34.44     -80.00
384      88.27      17.56     -80.00
385      90.00       0.0      -80.00
386      88.27     -17.56     -80.00
387      83.15     -34.44     -80.00
388      74.83     -50.00     -80.00
389      63.64     -63.64     -80.00
390      50.00     -74.83     -80.00
391      34.44     -83.15     -80.00
392      17.56     -88.27     -80.00
393       0.0      -90.00     -80.00
394     -17.56     -88.27     -80.00
395     -34.44     -83.15     -80.00
396     -50.00     -74.83     -80.00
397     -63.64     -63.64     -80.00
398     -74.83     -50.00     -80.00
399     -83.15     -34.44     -80.00
400     -88.27     -17.56     -80.00
401     -90.00       0.0      -80.00
402     -88.27      17.56     -80.00
403     -83.15      34.44     -80.00
404     -74.83      50.00     -80.00
405     -63.64      63.64     -80.00
406     -50.00      74.83     -80.00
407     -34.44      83.15     -80.00
408     -17.56      88.27     -80.00
409       0.0       90.00     -95.00
410      34.44      83.15     -95.00
411      63.64      63.64     -95.00
412      83.15      34.44     -95.00
413      90.00       0.0      -95.00
414      83.15     -34.44     -95.00
415      63.64     -63.64     -95.00
416      34.44     -83.15     -95.00
417       0.0      -90.00     -95.00
418     -34.44     -83.15     -95.00
419     -63.64     -63.64     -95.00
420     -83.15     -34.44     -95.00
421     -90.00       0.0      -95.00
422     -83.15      34.44     -95.00
423     -63.64      63.64     -95.00
424     -34.44      83.15     -95.00
425       0.0        0.0     -110.00
426       0.0       30.00    -110.00
427      21.21      21.21    -110.00
428      30.00       0.0     -110.00
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429	21.21	-21.21	-110.00
430	0.0	-30.00	-110.00
431	-21.21	-21.21	-110.00
432	-30.00	0.0	-110.00
433	-21.21	21.21	-110.00
434	0.0	60.00	-110.00
435	22.96	55.43	-110.00
436	42.43	42.43	-110.00
437	55.43	22.96	-110.00
438	60.00	0.0	-110.00
439	55.43	-22.96	-110.00
440	42.43	-42.43	-110.00
441	22.96	-55.43	-110.00
442	0.0	-60.00	-110.00
443	-22.96	-55.43	-110.00
444	-42.43	-42.43	-110.00
445	-55.43	-22.96	-110.00
446	-60.00	0.0	-110.00
447	-55.43	22.96	-110.00
448	-42.43	42.43	-110.00
449	-22.96	55.43	-110.00
450	0.0	75.00	-110.00
451	16.80	71.60	-110.00
452	38.10	62.90	-110.00
453	53.03	53.03	-110.00
454	62.80	38.60	-110.00
455	71.30	17.00	-110.00
456	75.00	0.0	-110.00
457	71.30	-17.00	-110.00
458	62.80	-38.60	-110.00
459	53.03	-53.03	-110.00
460	38.10	-62.90	-110.00
461	17.10	-71.80	-110.00
462	0.0	-75.00	-110.00
463	-16.90	-71.80	-110.00
464	-38.50	-62.80	-110.00
465	-53.03	-53.03	-110.00
466	-62.70	-38.60	-110.00
467	-71.30	-17.10	-110.00
468	-75.00	0.0	-110.00
469	-71.30	17.10	-110.00
470	-62.90	38.30	-110.00
471	-53.03	53.03	-110.00
472	-38.60	62.80	-110.00
473	-17.30	71.30	-110.00
474	0.0	90.00	-110.00
475	17.56	88.27	-110.00
476	34.44	83.15	-110.00

477	50.00	74.83	-110.00
478	63.64	63.64	-110.00
479	74.83	50.00	-110.00
480	83.15	34.44	-110.00
481	88.27	17.56	-110.00
482	90.00	0.0	-110.00
483	88.27	-17.56	-110.00
484	83.15	-34.44	-110.00
485	74.83	-50.00	-110.00
486	63.64	-63.64	-110.00
487	50.00	-74.83	-110.00
488	34.44	-83.15	-110.00
489	17.56	-88.27	-110.00
490	0.0	-90.00	-110.00
491	-17.56	-88.27	-110.00
492	-34.44	-83.15	-110.00
493	-50.00	-74.83	-110.00
494	-63.64	-63.64	-110.00
495	-74.83	-50.00	-110.00
496	-83.15	-34.44	-110.00
497	-88.27	-17.56	-110.00
498	-90.00	0.0	-110.00
499	-88.27	17.56	-110.00
500	-83.15	34.44	-110.00
501	-74.83	50.00	-110.00
502	-63.64	63.64	-110.00
503	-50.00	74.83	-110.00
504	-34.44	83.15	-110.00
505	-17.56	88.27	-110.00

APPENDIX B

MESH II CONNECTIVITY AND COORDINATES

The connectivity matrix for the 192-element mesh is as follows:

ELEMENT NUMBER	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	3	12	11	10	2	26	28	27	35	37	46	45	44	36
2	1	4	14	13	12	3	26	29	28	35	38	48	47	46	37
3	1	5	16	15	14	4	26	30	29	35	39	50	49	48	38
4	1	6	18	17	16	5	26	31	30	35	40	52	51	50	39
5	1	7	20	19	18	6	26	32	31	35	41	54	53	52	40
6	1	8	22	21	20	7	26	33	32	35	42	56	55	54	41
7	1	9	24	23	22	8	26	34	33	35	43	58	57	56	42
8	1	2	10	25	24	9	26	27	34	35	36	44	59	58	43
9	35	37	46	45	44	36	60	62	61	69	71	80	79	78	70
10	35	38	48	47	46	37	60	63	62	69	72	82	81	80	71
11	35	39	50	49	48	38	60	64	63	69	73	84	83	82	72
12	35	40	52	51	50	39	60	65	64	69	74	86	85	84	73
13	35	41	54	53	52	40	60	66	65	69	75	88	87	86	74
14	35	42	56	55	54	41	60	67	66	69	76	90	89	88	75
15	35	43	58	57	56	42	60	68	67	69	77	92	91	90	76
16	35	36	44	59	58	43	60	61	68	69	70	78	93	92	77
17	69	71	80	79	78	70	94	96	95	103	105	114	113	112	104
18	69	72	82	81	80	71	94	97	96	103	106	116	115	114	105
19	69	73	84	83	82	72	94	98	97	103	107	118	117	116	106
20	69	74	86	85	84	73	94	99	98	103	108	120	119	118	107
21	69	75	88	87	86	74	94	100	99	103	109	122	121	120	108
22	69	76	90	89	88	75	94	101	100	103	110	124	123	122	109
23	69	77	92	91	90	76	94	102	101	103	111	126	125	124	110
24	69	70	78	93	92	77	94	95	102	103	104	112	127	126	111
25	103	105	114	113	112	104	128	130	129	137	139	148	147	146	138
26	103	106	116	115	114	105	128	131	130	137	140	150	149	148	139
27	103	107	118	117	116	106	128	132	131	137	141	152	151	150	140
28	103	108	120	119	118	107	128	133	132	137	142	154	153	152	141
29	103	109	122	121	120	108	128	134	133	137	143	156	155	154	142
30	103	110	124	123	122	109	128	135	134	137	144	158	157	156	143
31	103	111	126	125	124	110	128	136	135	137	145	160	159	158	144
32	103	104	112	127	126	111	128	129	136	137	138	146	161	160	145
33	137	139	148	147	146	138	162	164	163	300	302	311	310	309	301
34	137	140	150	149	148	139	162	165	164	300	303	313	312	311	302
35	137	141	152	151	150	140	162	165	165	300	304	315	314	313	303

36	137	142	154	153	152	141	162	167	166	300	305	317	316	315	304
37	137	143	156	155	154	142	162	168	167	300	306	319	318	317	305
38	137	144	158	157	156	143	162	169	168	300	307	321	320	319	306
39	137	145	160	159	158	144	162	170	169	300	308	323	322	321	307
40	137	138	146	161	160	145	162	163	170	300	301	309	324	323	308
41	717	710	695	694	693	709	171	173	172	1	3	12	11	10	2
42	717	711	697	696	695	710	171	174	173	1	4	14	13	12	3
43	717	712	699	698	697	711	171	175	174	1	5	16	15	14	4
44	717	713	701	700	699	712	171	176	175	1	6	18	17	16	5
45	717	714	703	702	701	713	171	177	176	1	7	20	19	18	6
46	717	715	705	704	703	714	171	178	177	1	8	22	21	20	7
47	717	716	707	706	705	715	171	179	178	1	9	24	23	22	8
48	717	709	693	708	707	716	171	172	179	1	2	10	25	24	9
49	693	670	639	638	637	669	172	622	621	10	181	591	590	589	180
50	639	670	693	694	695	671	622	172	173	591	181	10	11	12	182
51	695	672	641	640	639	671	173	623	622	12	183	593	592	591	182
52	695	673	643	642	641	672	173	624	623	12	184	595	594	593	183
53	643	673	695	696	697	674	624	173	174	595	184	12	13	14	185
54	697	675	645	644	643	674	174	625	624	14	186	597	596	595	185
55	697	676	647	646	645	675	174	625	625	14	187	599	598	597	186
56	647	676	697	698	699	677	626	174	175	599	187	14	15	16	188
57	699	678	649	648	647	677	175	627	626	16	189	601	600	599	188
58	699	679	651	650	649	678	175	628	627	16	190	603	602	601	189
59	651	679	699	700	701	680	628	175	176	603	190	16	17	18	191
60	701	681	653	652	651	680	176	629	628	18	192	605	604	603	191
61	701	682	655	654	653	681	176	630	629	18	193	607	606	605	192
62	655	682	701	702	703	683	630	176	177	607	193	18	19	20	194
63	703	684	657	656	655	683	177	631	630	20	195	609	608	607	194
64	703	685	659	658	657	684	177	632	631	20	196	611	610	609	195
65	659	685	703	704	705	686	632	177	178	611	196	20	21	22	197
66	705	687	661	660	659	686	178	633	632	22	198	613	612	611	197
67	705	688	663	662	661	687	178	634	633	22	199	615	614	613	198
68	663	688	705	706	707	689	634	178	179	615	199	22	23	24	200
69	707	690	665	664	663	689	179	635	634	24	201	617	616	615	200
70	707	691	667	666	665	690	179	636	635	24	202	619	618	617	201
71	667	691	707	708	693	692	636	179	172	619	202	24	25	10	203
72	693	669	637	668	667	692	172	621	636	10	180	589	620	619	203
73	10	181	591	590	589	180	27	574	573	44	205	543	542	541	204
74	591	181	10	11	12	182	574	27	28	543	205	44	45	46	206
75	12	183	593	592	591	182	28	575	574	46	207	545	544	543	206
76	12	184	595	594	593	183	28	576	575	46	208	547	546	545	207
77	595	184	12	13	14	185	576	28	29	547	208	46	47	48	209
78	14	186	597	596	595	185	29	577	576	48	210	549	548	547	209
79	14	187	599	598	597	186	29	578	577	48	211	551	550	549	210
80	599	187	14	15	16	188	578	29	30	551	211	48	49	50	212
81	16	189	601	600	599	188	30	579	578	50	213	553	552	551	212
82	16	190	603	602	601	189	30	580	579	50	214	555	554	553	213
83	603	190	16	17	18	191	580	30	31	555	214	50	51	52	215

84	18	192	605	604	603	191	31	581	580	52	216	557	556	555	215
85	18	193	607	606	605	192	31	582	581	52	217	559	558	557	216
86	607	193	13	19	20	194	582	31	32	559	217	52	53	54	218
87	20	195	609	608	607	194	32	583	582	54	219	561	560	559	218
88	20	196	611	610	609	195	32	584	583	54	220	563	562	561	219
89	611	196	20	21	22	197	584	32	33	563	220	54	55	56	221
90	22	198	613	612	611	197	33	585	584	56	222	565	564	563	221
91	22	199	615	614	613	198	33	586	585	56	223	567	566	565	222
92	615	199	22	23	24	200	586	33	34	567	223	56	57	58	224
93	24	201	617	616	615	200	34	587	586	58	225	569	568	567	224
94	24	202	619	618	617	201	34	588	587	58	226	571	570	569	225
95	619	202	24	25	10	203	588	34	27	571	226	58	59	44	227
96	10	180	589	620	619	203	27	573	588	44	204	541	572	571	227
97	44	205	543	542	541	204	61	525	525	78	229	495	494	493	228
98	543	205	44	45	46	206	526	61	62	495	229	78	79	80	230
99	46	207	545	544	543	206	62	527	526	80	231	497	496	495	230
100	46	208	547	546	545	207	62	528	527	80	232	499	498	497	231
101	547	208	46	47	48	209	528	62	63	499	232	80	81	82	233
102	48	210	549	548	547	209	63	529	528	82	234	501	500	499	233
103	48	211	551	550	549	210	63	530	529	82	235	503	502	501	234
104	551	211	48	49	50	212	530	63	64	503	235	82	83	84	236
105	50	213	553	552	551	212	64	531	530	84	237	505	504	503	236
106	50	214	555	554	553	213	64	532	531	84	238	507	506	505	237
107	555	214	50	51	52	215	532	64	65	507	238	84	85	86	239
108	52	216	557	556	555	215	65	533	532	86	240	509	508	507	239
109	52	217	559	558	557	216	65	534	533	86	241	511	510	509	240
110	559	217	52	53	54	218	534	65	66	511	241	86	87	88	242
111	54	219	561	560	559	218	66	535	534	88	243	513	512	511	242
112	54	220	563	562	561	219	66	536	535	88	244	515	514	513	243
113	563	220	54	55	56	221	536	65	67	515	244	88	89	90	245
114	56	222	565	564	563	221	67	537	536	90	246	517	516	515	245
115	56	223	567	566	565	222	67	538	537	90	247	519	518	517	246
116	567	223	56	57	58	224	538	67	68	519	247	90	91	92	248
117	58	225	569	568	567	224	68	539	538	92	249	521	520	519	248
118	58	226	571	570	569	225	68	540	539	92	250	523	522	521	249
119	571	226	58	59	44	227	540	68	61	523	250	92	93	78	251
120	44	204	541	572	571	227	61	525	540	78	228	493	524	523	251
121	78	229	495	494	493	228	95	478	477	112	253	447	446	445	252
122	495	229	78	79	80	230	478	95	96	447	253	112	113	114	254
123	80	231	497	496	495	230	96	479	478	114	255	449	448	447	254
124	80	232	499	498	497	231	96	480	479	114	256	451	450	449	255
125	499	232	80	81	82	233	480	96	97	451	256	114	115	116	257
126	82	234	501	500	499	233	97	481	480	116	258	453	452	451	257
127	82	235	503	502	501	234	97	482	481	116	259	455	454	453	258
128	503	235	82	83	84	236	482	97	98	455	259	116	117	118	260
129	84	237	505	504	503	236	98	483	482	118	261	457	456	455	260
130	84	238	507	506	505	237	98	484	483	118	262	459	458	457	261
131	507	238	84	85	86	239	484	98	99	459	262	118	119	120	263

132	86	240	509	508	507	239	99	485	484	120	264	461	460	459	263
133	86	241	511	510	509	240	99	486	485	120	265	463	462	461	264
134	511	241	86	87	88	242	486	99	100	463	265	120	121	122	266
135	88	243	513	512	511	242	100	487	486	122	267	465	464	463	266
136	88	244	515	514	513	243	100	488	487	122	268	467	466	465	267
137	515	244	88	89	90	245	488	100	101	467	268	122	123	124	269
138	90	246	517	516	515	245	101	489	488	124	270	469	468	467	269
139	90	247	519	518	517	246	101	490	489	124	271	471	470	469	270
140	519	247	90	91	92	248	490	101	102	471	271	124	125	126	272
141	92	249	521	520	519	248	102	491	490	126	273	473	472	471	272
142	92	250	523	522	521	249	102	492	491	126	274	475	474	473	273
143	523	250	92	93	78	251	492	102	95	475	274	126	127	112	275
144	78	228	493	524	523	251	95	477	492	112	252	445	476	475	275
145	112	253	447	446	445	252	129	430	429	146	277	399	398	397	276
146	447	253	112	113	114	254	430	129	130	399	277	146	147	148	278
147	114	255	449	448	447	254	130	431	430	148	279	401	400	399	278
148	114	256	451	450	449	255	130	432	431	148	280	403	402	401	279
149	451	256	114	115	116	257	432	130	131	403	280	148	149	150	281
150	116	258	453	452	451	257	131	433	432	150	282	405	404	403	281
151	116	259	455	454	453	258	131	434	433	150	283	407	406	405	282
152	455	259	116	117	118	260	434	131	132	407	283	150	151	152	284
153	118	261	457	456	455	260	132	435	434	152	285	409	408	407	284
154	118	262	459	458	457	261	132	436	435	152	286	411	410	409	285
155	459	262	118	119	120	263	436	132	133	411	286	152	153	154	287
156	120	264	461	460	459	263	133	437	436	154	288	413	412	411	287
157	120	265	463	462	461	264	133	438	437	154	289	415	414	413	288
158	463	265	120	121	122	266	438	133	134	415	289	154	155	156	290
159	122	267	465	464	463	266	134	439	438	156	291	417	416	415	290
160	122	268	467	466	465	267	134	440	439	156	292	419	418	417	291
161	467	268	122	123	124	269	440	134	135	419	292	156	157	158	293
162	124	270	469	468	467	269	135	441	440	158	294	421	420	419	293
163	124	271	471	470	469	270	135	442	441	158	295	423	422	421	294
164	471	271	124	125	126	272	442	135	136	423	295	158	159	160	296
165	126	273	473	472	471	272	136	443	442	160	297	425	424	423	296
166	126	274	475	474	473	273	136	444	443	160	298	427	426	425	297
167	475	274	126	127	112	275	444	136	129	427	298	160	161	146	299
168	112	252	445	476	475	275	129	429	444	146	276	397	428	427	299
169	146	277	399	398	397	276	163	382	381	309	326	351	350	349	325
170	399	277	146	147	148	278	382	163	164	351	326	309	310	311	327
171	148	279	401	400	399	278	164	383	382	311	328	353	352	351	327
172	148	280	403	402	401	279	164	384	383	311	329	355	354	353	328
173	403	280	148	149	150	281	384	164	165	355	329	311	312	313	330
174	150	282	405	404	403	281	165	385	384	313	331	357	356	355	330
175	150	283	407	406	405	282	165	386	385	313	332	359	358	357	331
176	407	283	150	151	152	284	386	165	166	359	332	313	314	315	333
177	152	285	409	408	407	284	166	387	386	315	334	361	360	359	333
178	152	286	411	410	409	285	166	388	387	315	335	363	362	361	334
179	411	286	152	153	154	287	388	166	167	363	335	315	316	317	336

180	154	288	413	412	411	287	167	389	388	317	337	365	364	363	336
181	154	289	415	414	413	288	167	390	389	317	338	367	366	365	337
182	415	289	154	155	156	290	390	167	168	367	338	317	318	319	339
183	156	291	417	416	415	290	168	391	390	319	340	369	368	367	339
184	156	292	419	418	417	291	168	392	391	319	341	371	370	369	340
185	419	292	156	157	158	293	392	168	169	371	341	319	320	321	342
186	158	294	421	420	419	293	169	393	392	321	343	373	372	371	342
187	158	295	423	422	421	294	169	394	393	321	344	375	374	373	343
188	423	295	158	159	160	296	394	169	170	375	344	321	322	323	345
189	160	297	425	424	423	296	170	395	394	323	346	377	376	375	345
190	160	298	427	426	425	297	170	396	395	323	347	379	378	377	346
191	427	298	160	161	146	299	396	170	163	379	347	323	324	309	348
192	146	276	397	428	427	299	163	381	396	309	325	349	380	379	348

The coordinates of the 192-element mesh are as follows:

NODE NUMBER	X	Y	Z
1	0.0	0.0	80.00
2	0.0	30.00	80.00
3	21.21	21.21	80.00
4	30.00	0.0	80.00
5	21.21	-21.21	80.00
6	0.0	-30.00	80.00
7	-21.21	-21.21	80.00
8	-30.00	0.0	80.00
9	-21.21	21.21	80.00
10	0.0	60.00	80.00
11	22.96	55.43	80.00
12	42.43	42.43	80.00
13	55.43	22.96	80.00
14	60.00	0.0	80.00
15	55.43	-22.96	80.00
16	42.43	-42.43	80.00
17	22.96	-55.43	80.00
18	0.0	-60.00	80.00
19	-22.96	-55.43	80.00
20	-42.43	-42.43	80.00
21	-55.43	-22.96	80.00
22	-60.00	0.0	80.00
23	-55.43	22.96	80.00
24	-42.43	42.43	80.00
25	-22.96	55.43	80.00
26	0.0	0.0	60.00

27	0.0	60.00	60.00
28	42.43	42.43	60.00
29	60.00	0.0	60.00
30	42.43	-42.43	60.00
31	0.0	-60.00	60.00
32	-42.42	-42.43	60.00
33	-60.00	0.0	60.00
34	-42.43	42.43	60.00
35	0.0	0.0	40.00
36	0.0	30.00	40.00
37	21.21	21.21	40.00
38	30.00	0.0	40.00
39	21.21	-21.21	40.00
40	0.0	-30.00	40.00
41	-21.21	-21.21	40.00
42	-30.00	0.0	40.00
43	-21.21	21.21	40.00
44	0.0	60.00	40.00
45	22.96	55.43	40.00
46	42.43	42.43	40.00
47	55.43	22.96	40.00
48	60.00	0.0	40.00
49	55.43	-22.96	40.00
50	42.43	-42.43	40.00
51	22.96	-55.43	40.00
52	0.0	-60.00	40.00
53	-22.96	-55.43	40.00
54	-42.43	-42.43	40.00
55	-55.43	-22.96	40.00
56	-60.00	0.0	40.00
57	-55.43	22.96	40.00
58	-42.43	42.43	40.00
59	-22.96	55.43	40.00
60	0.0	0.0	20.00
61	0.0	60.00	20.00
62	42.43	42.43	20.00
63	60.00	0.0	20.00
64	42.43	-42.43	20.00
65	0.0	-60.00	20.00
66	-42.42	-42.43	20.00
67	-60.00	0.0	20.00
68	-42.43	42.43	20.00
69	0.0	0.0	0.0
70	0.0	30.00	0.0
71	21.21	21.21	0.0
72	30.00	0.0	0.0
73	21.21	-21.21	0.0
74	0.0	-30.00	0.0

75	-21.21	-21.21	0.0
76	-30.00	0.0	0.0
77	-21.21	21.21	0.0
78	0.0	60.00	0.0
79	22.96	55.43	0.0
80	42.43	42.43	0.0
81	55.43	22.96	0.0
82	60.00	0.0	0.0
83	55.43	-22.96	0.0
84	42.43	-42.43	0.0
85	22.96	-55.43	0.0
86	0.0	-60.00	0.0
87	-22.96	-55.43	0.0
88	-42.43	-42.43	0.0
89	-55.43	-22.96	0.0
90	-60.00	0.0	0.0
91	-55.43	22.96	0.0
92	-42.43	42.43	0.0
93	-22.96	55.43	0.0
94	0.0	0.0	-20.00
95	0.0	60.00	-20.00
96	42.43	42.43	-20.00
97	60.00	0.0	-20.00
98	42.43	-42.43	-20.00
99	0.0	-60.00	-20.00
100	-42.42	-42.43	-20.00
101	-60.00	0.0	-20.00
102	-42.43	42.43	-20.00
103	0.0	0.0	-40.00
104	0.0	30.00	-40.00
105	21.21	21.21	-40.00
106	30.00	0.0	-40.00
107	21.21	-21.21	-40.00
108	0.0	-30.00	-40.00
109	-21.21	-21.21	-40.00
110	-30.00	0.0	-40.00
111	-21.21	21.21	-40.00
112	0.0	60.00	-40.00
113	22.96	55.43	-40.00
114	42.43	42.43	-40.00
115	55.43	22.96	-40.00
116	60.00	0.0	-40.00
117	55.43	-22.96	-40.00
118	42.43	-42.43	-40.00
119	22.96	-55.43	-40.00
120	0.0	-60.00	-40.00
121	-22.96	-55.43	-40.00
122	-42.43	-42.43	-40.00

123	-55.43	-22.96	-40.00
124	-60.00	0.0	-40.00
125	-55.43	22.96	-40.00
126	-42.43	42.43	-40.00
127	-22.96	55.43	-40.00
128	0.0	0.0	-60.00
129	0.0	60.00	-60.00
130	42.43	42.43	-60.00
131	60.00	0.0	-60.00
132	42.43	-42.43	-60.00
133	0.0	-60.00	-60.00
134	-42.42	-42.43	-60.00
135	-60.00	0.0	-60.00
136	-42.43	42.43	-60.00
137	0.0	0.0	-80.00
138	0.0	30.00	-80.00
139	21.21	21.21	-80.00
140	30.00	0.0	-80.00
141	21.21	-21.21	-80.00
142	0.0	-30.00	-80.00
143	-21.21	-21.21	-80.00
144	-30.00	0.0	-80.00
145	-21.21	21.21	-80.00
146	0.0	60.00	-80.00
147	22.96	55.43	-80.00
148	42.43	42.43	-80.00
149	55.43	22.96	-80.00
150	60.00	0.0	-80.00
151	55.43	-22.96	-80.00
152	42.43	-42.43	-80.00
153	22.96	-55.43	-80.00
154	0.0	-60.00	-80.00
155	-22.96	-55.43	-80.00
156	-42.43	-42.43	-80.00
157	-55.43	-22.96	-80.00
158	-60.00	0.0	-80.00
159	-55.43	22.96	-80.00
160	-42.43	42.43	-80.00
161	-22.96	55.43	-80.00
162	0.0	0.0	-95.00
163	0.0	60.00	-95.00
164	42.43	42.43	-95.00
165	60.00	0.0	-95.00
166	42.43	-42.43	-95.00
167	0.0	-60.00	-95.00
168	-42.42	-42.43	-95.00
169	-60.00	0.0	-95.00
170	-42.43	42.43	-95.00

171	0.0	0.0	95.00
172	0.0	60.00	95.00
173	42.43	42.43	95.00
174	60.00	0.0	95.00
175	42.43	-42.43	95.00
176	0.0	-60.00	95.00
177	-42.42	-42.43	95.00
178	-60.00	0.0	95.00
179	-42.43	42.43	95.00
180	0.0	75.00	80.00
181	16.80	71.60	80.00
182	38.10	62.90	80.00
183	53.03	53.03	80.00
184	62.80	38.60	80.00
185	71.30	17.00	80.00
186	75.00	0.0	80.00
187	71.30	-17.00	80.00
188	62.80	-38.60	80.00
189	53.03	-53.03	80.00
190	38.10	-62.90	80.00
191	17.10	-71.80	80.00
192	0.0	-75.00	80.00
193	-16.90	-71.80	80.00
194	-38.50	-62.80	80.00
195	-53.03	-53.03	80.00
196	-62.70	-38.60	80.00
197	-71.30	-17.10	80.00
198	-75.00	0.0	80.00
199	-71.30	17.10	80.00
200	-62.90	38.30	80.00
201	-53.03	53.03	80.00
202	-38.60	62.80	80.00
203	-17.30	71.30	80.00
204	0.0	75.00	40.00
205	16.80	71.60	40.00
206	38.10	62.90	40.00
207	53.03	53.03	40.00
208	62.80	38.60	40.00
209	71.30	17.00	40.00
210	75.00	0.0	40.00
211	71.30	-17.00	40.00
212	62.80	-38.60	40.00
213	53.03	-53.03	40.00
214	38.10	-62.90	40.00
215	17.10	-71.80	40.00
216	0.0	-75.00	40.00
217	-16.90	-71.80	40.00
218	-38.50	-62.80	40.00

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219    -53.03    -53.03     40.00
220    -62.70    -38.60     40.00
221    -71.30    -17.10     40.00
222    -75.00      0.0      40.00
223    -71.30     17.10     40.00
224    -62.90     38.30     40.00
225    -53.03     53.03     40.00
226    -38.60     62.80     40.00
227    -17.30     71.30     40.00
228      0.0      75.00      0.0
229     16.80     71.60      0.0
230     38.10     62.90      0.0
231     53.03     53.03      0.0
232     62.80     38.60      0.0
233     71.30     17.00      0.0
234     75.00      0.0       0.0
235     71.30    -17.00      0.0
236     62.80    -38.60      0.0
237     53.03    -53.03      0.0
238     38.10    -62.90      0.0
239     17.10    -71.80      0.0
240      0.0     -75.00      0.0
241    -16.90    -71.80      0.0
242    -38.50    -62.80      0.0
243    -53.03    -53.03      0.0
244    -62.70    -38.60      0.0
245    -71.30    -17.10      0.0
246    -75.00      0.0       0.0
247    -71.30     17.10      0.0
248    -62.90     38.30      0.0
249    -53.03     53.03      0.0
250    -38.60     62.80      0.0
251    -17.30     71.30      0.0
252      0.0      75.00    -40.00
253     16.80     71.60    -40.00
254     38.10     62.90    -40.00
255     53.03     53.03    -40.00
256     62.80     38.60    -40.00
257     71.30     17.00    -40.00
258     75.00      0.0     -40.00
259     71.30    -17.00    -40.00
260     62.80    -38.60    -40.00
261     53.03    -53.03    -40.00
262     38.10    -62.90    -40.00
263     17.10    -71.80    -40.00
264      0.0     -75.00    -40.00
265    -16.90    -71.80    -40.00
266    -38.50    -62.80    -40.00
```

267	-53.03	-53.03	-40.00
268	-62.70	-38.60	-40.00
269	-71.30	-17.10	-40.00
270	-75.00	0.0	-40.00
271	-71.30	17.10	-40.00
272	-62.90	38.30	-40.00
273	-53.03	53.03	-40.00
274	-38.60	62.80	-40.00
275	-17.30	71.30	-40.00
276	0.0	75.00	-80.00
277	16.80	71.60	-80.00
278	38.10	62.90	-80.00
279	53.03	53.03	-80.00
280	62.80	38.60	-80.00
281	71.30	17.00	-80.00
282	75.00	0.0	-80.00
283	71.30	-17.00	-80.00
284	62.80	-38.60	-80.00
285	53.03	-53.03	-80.00
286	38.10	-62.90	-80.00
287	17.10	-71.80	-80.00
288	0.0	-75.00	-80.00
289	-16.90	-71.80	-80.00
290	-38.50	-62.80	-80.00
291	-53.03	-53.03	-80.00
292	-62.70	-38.60	-80.00
293	-71.30	-17.10	-80.00
294	-75.00	0.0	-80.00
295	-71.30	17.10	-80.00
296	-62.90	38.30	-80.00
297	-53.03	53.03	-80.00
298	-38.60	62.80	-80.00
299	-17.30	71.30	-80.00
300	0.0	0.0	-110.00
301	0.0	30.00	-110.00
302	21.21	21.21	-110.00
303	30.00	0.0	-110.00
304	21.21	-21.21	-110.00
305	0.0	-30.00	-110.00
306	-21.21	-21.21	-110.00
307	-30.00	0.0	-110.00
308	-21.21	21.21	-110.00
309	0.0	60.00	-110.00
310	22.96	55.43	-110.00
311	42.43	42.43	-110.00
312	55.43	22.96	-110.00
313	60.00	0.0	-110.00
314	55.43	-22.96	-110.00

315	42.43	-42.43	-110.00
316	22.96	-55.43	-110.00
317	0.0	-60.00	-110.00
318	-22.96	-55.43	-110.00
319	-42.43	-42.43	-110.00
320	-55.43	-22.96	-110.00
321	-60.00	0.0	-110.00
322	-55.43	22.96	-110.00
323	-42.43	42.43	-110.00
324	-22.96	55.43	-110.00
325	0.0	75.00	-110.00
326	16.80	71.60	-110.00
327	38.10	62.90	-110.00
328	53.03	53.03	-110.00
329	62.80	38.60	-110.00
330	71.30	17.00	-110.00
331	75.00	0.0	-110.00
332	71.30	-17.00	-110.00
333	62.80	-38.60	-110.00
334	53.03	-53.03	-110.00
335	38.10	-62.90	-110.00
336	17.10	-71.80	-110.00
337	0.0	-75.00	-110.00
338	-16.90	-71.80	-110.00
339	-38.50	-62.80	-110.00
340	-53.03	-53.03	-110.00
341	-62.70	-38.60	-110.00
342	-71.30	-17.10	-110.00
343	-75.00	0.0	-110.00
344	-71.30	17.10	-110.00
345	-62.90	38.30	-110.00
346	-53.03	53.03	-110.00
347	-38.60	62.80	-110.00
348	-17.30	71.30	-110.00
349	0.0	90.00	-110.00
350	17.56	88.27	-110.00
351	34.44	83.15	-110.00
352	50.00	74.83	-110.00
353	63.64	63.64	-110.00
354	74.83	50.00	-110.00
355	83.15	34.44	-110.00
356	88.27	17.56	-110.00
357	90.00	0.0	-110.00
358	88.27	-17.56	-110.00
359	83.15	-34.44	-110.00
360	74.83	-50.00	-110.00
361	63.64	-63.64	-110.00
362	50.00	-74.83	-110.00

363	34.44	-83.15	-110.00
364	17.56	-88.27	-110.00
365	0.0	-90.00	-110.00
366	-17.56	-88.27	-110.00
367	-34.44	-83.15	-110.00
368	-50.00	-74.83	-110.00
369	-63.64	-63.64	-110.00
370	-74.83	-50.00	-110.00
371	-83.15	-34.44	-110.00
372	-88.27	-17.56	-110.00
373	-90.00	0.0	-110.00
374	-88.27	17.56	-110.00
375	-83.15	34.44	-110.00
376	-74.83	50.00	-110.00
377	-63.64	63.64	-110.00
378	-50.00	74.83	-110.00
379	-34.44	83.15	-110.00
380	-17.56	88.27	-110.00
381	0.0	90.00	-95.00
382	34.44	83.15	-95.00
383	63.64	63.64	-95.00
384	83.15	34.44	-95.00
385	90.00	0.0	-95.00
386	83.15	-34.44	-95.00
387	63.64	-63.64	-95.00
388	34.44	-83.15	-95.00
389	0.0	-90.00	-95.00
390	-34.44	-83.15	-95.00
391	-63.64	-63.64	-95.00
392	-83.15	-34.44	-95.00
393	-90.00	0.0	-95.00
394	-83.15	34.44	-95.00
395	-63.64	63.64	-95.00
396	-34.44	83.15	-95.00
397	0.0	90.00	-80.00
398	17.56	88.27	-80.00
399	34.44	83.15	-80.00
400	50.00	74.83	-80.00
401	63.64	63.64	-80.00
402	74.83	50.00	-80.00
403	83.15	34.44	-80.00
404	88.27	17.56	-80.00
405	90.00	0.0	-80.00
406	88.27	-17.56	-80.00
407	83.15	-34.44	-80.00
408	74.83	-50.00	-80.00
409	63.64	-63.64	-80.00
410	50.00	-74.83	-80.00

411	34.44	-83.15	-80.00
412	17.56	-88.27	-80.00
413	0.0	-90.00	-80.00
414	-17.56	-88.27	-80.00
415	-34.44	-83.15	-80.00
416	-50.00	-74.83	-80.00
417	-63.64	-63.64	-80.00
418	-74.83	-50.00	-80.00
419	-83.15	-34.44	-80.00
420	-88.27	-17.56	-80.00
421	-90.00	0.0	-80.00
422	-88.27	17.56	-80.00
423	-83.15	34.44	-80.00
424	-74.83	50.00	-80.00
425	-63.64	63.64	-80.00
426	-50.00	74.83	-80.00
427	-34.44	83.15	-80.00
428	-17.56	88.27	-80.00
429	0.0	90.00	-60.00
430	34.44	83.15	-60.00
431	63.64	63.64	-60.00
432	83.15	34.44	-60.00
433	90.00	0.0	-60.00
434	83.15	-34.44	-60.00
435	63.64	-63.64	-60.00
436	34.44	-83.15	-60.00
437	0.0	-90.00	-60.00
438	-34.44	-83.15	-60.00
439	-63.64	-63.64	-60.00
440	-83.15	-34.44	-60.00
441	-90.00	0.0	-60.00
442	-83.15	34.44	-60.00
443	-63.64	63.64	-60.00
444	-34.44	83.15	-60.00
445	0.0	90.00	-40.00
446	17.56	88.27	-40.00
447	34.44	83.15	-40.00
448	50.00	74.83	-40.00
449	63.64	63.64	-40.00
450	74.83	50.00	-40.00
451	83.15	34.44	-40.00
452	88.27	17.56	-40.00
453	90.00	0.0	-40.00
454	88.27	-17.56	-40.00
455	83.15	-34.44	-40.00
456	74.83	-50.00	-40.00
457	63.64	-63.64	-40.00
458	50.00	-74.83	-40.00

459	34.44	-83.15	-40.00
460	17.56	-88.27	-40.00
461	0.0	-90.00	-40.00
462	-17.56	-88.27	-40.00
463	-34.44	-83.15	-40.00
464	-50.00	-74.83	-40.00
465	-63.64	-63.64	-40.00
466	-74.83	-50.00	-40.00
467	-83.15	-34.44	-40.00
468	-88.27	-17.56	-40.00
469	-90.00	0.0	-40.00
470	-88.27	17.56	-40.00
471	-83.15	34.44	-40.00
472	-74.83	50.00	-40.00
473	-63.64	63.64	-40.00
474	-50.00	74.83	-40.00
475	-34.44	83.15	-40.00
476	-17.56	88.27	-40.00
477	0.0	90.00	-20.00
478	34.44	83.15	-20.00
479	63.64	63.64	-20.00
480	83.15	34.44	-20.00
481	90.00	0.0	-20.00
482	83.15	-34.44	-20.00
483	63.64	-63.64	-20.00
484	34.44	-83.15	-20.00
485	0.0	-90.00	-20.00
486	-34.44	-83.15	-20.00
487	-63.64	-63.64	-20.00
488	-83.15	-34.44	-20.00
489	-90.00	0.0	-20.00
490	-83.15	34.44	-20.00
491	-63.64	63.64	-20.00
492	-34.44	83.15	-20.00
493	0.0	90.00	0.0
494	17.56	88.27	0.0
495	34.44	83.15	0.0
496	50.00	74.83	0.0
497	63.64	63.64	0.0
498	74.83	50.00	0.0
499	83.15	34.44	0.0
500	88.27	17.56	0.0
501	90.00	0.0	0.0
502	88.27	-17.56	0.0
503	83.15	-34.44	0.0
504	74.83	-50.00	0.0
505	63.64	-63.64	0.0
506	50.00	-74.83	0.0

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507      34.44     -83.15      0.0
508      17.56     -88.27      0.0
509       0.0      -90.00      0.0
510     -17.56     -88.27      0.0
511     -34.44     -83.15      0.0
512     -50.00     -74.83      0.0
513     -63.64     -63.64      0.0
514     -74.83     -50.00      0.0
515     -83.15     -34.44      0.0
516     -88.27     -17.56      0.0
517     -90.00       0.0       0.0
518     -88.27      17.56      0.0
519     -83.15      34.44      0.0
520     -74.83      50.00      0.0
521     -63.64      63.64      0.0
522     -50.00      74.83      0.0
523     -34.44      83.15      0.0
524     -17.56      88.27      0.0
525       0.0       90.00     20.00
526      34.44      83.15     20.00
527      63.64      63.64     20.00
528      83.15      34.44     20.00
529      90.00       0.0      20.00
530      83.15     -34.44     20.00
531      63.64     -63.64     20.00
532      34.44     -83.15     20.00
533       0.0      -90.00     20.00
534     -34.44     -83.15     20.00
535     -63.64     -63.64     20.00
536     -83.15     -34.44     20.00
537     -90.00       0.0      20.00
538     -83.15      34.44     20.00
539     -63.64      63.64     20.00
540     -34.44      83.15     20.00
541       0.0       90.00     40.00
542      17.56      88.27     40.00
543      34.44      83.15     40.00
544      50.00      74.83     40.00
545      63.64      63.64     40.00
546      74.83      50.00     40.00
547      83.15      34.44     40.00
548      88.27      17.56     40.00
549      90.00       0.0      40.00
550      88.27     -17.56     40.00
551      83.15     -34.44     40.00
552      74.83     -50.00     40.00
553      63.64     -63.64     40.00
554      50.00     -74.83     40.00
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555     34.44     -83.15     40.00
556     17.56     -88.27     40.00
557      0.0      -90.00     40.00
558    -17.56     -88.27     40.00
559    -34.44     -83.15     40.00
560    -50.00     -74.83     40.00
561    -63.64     -63.64     40.00
562    -74.83     -50.00     40.00
563    -83.15     -34.44     40.00
564    -88.27     -17.56     40.00
565    -90.00       0.0      40.00
566    -88.27      17.56     40.00
567    -83.15      34.44     40.00
568    -74.83      50.00     40.00
569    -63.64      63.64     40.00
570    -50.00      74.83     40.00
571    -34.44      83.15     40.00
572    -17.56      88.27     40.00
573      0.0       90.00     60.00
574     34.44      83.15     60.00
575     63.64      63.64     60.00
576     83.15      34.44     60.00
577     90.00       0.0      60.00
578     83.15     -34.44     60.00
579     63.64     -63.64     60.00
580     34.44     -83.15     60.00
581      0.0      -90.00     60.00
582    -34.44     -83.15     60.00
583    -63.64     -63.64     60.00
584    -83.15     -34.44     60.00
585    -90.00       0.0      60.00
586    -83.15      34.44     60.00
587    -63.64      63.64     60.00
588    -34.44      83.15     60.00
589      0.0       90.00     80.00
590     17.56      88.27     80.00
591     34.44      83.15     80.00
592     50.00      74.83     80.00
593     63.64      63.64     80.00
594     74.83      50.00     80.00
595     83.15      34.44     80.00
596     88.27      17.56     80.00
597     90.00       0.0      80.00
598     88.27     -17.56     80.00
599     83.15     -34.44     80.00
600     74.83     -50.00     80.00
601     63.64     -63.64     80.00
602     50.00     -74.83     80.00
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603	34.44	-83.15	80.00
604	17.56	-88.27	80.00
605	0.0	-90.00	80.00
606	-17.56	-88.27	80.00
607	-34.44	-83.15	80.00
608	-50.00	-74.83	80.00
609	-63.64	-63.64	80.00
610	-74.83	-50.00	80.00
611	-83.15	-34.44	80.00
612	-88.27	-17.56	80.00
613	-90.00	0.0	80.00
614	-88.27	17.56	80.00
615	-83.15	34.44	80.00
616	-74.83	50.00	80.00
617	-63.64	63.64	80.00
618	-50.00	74.83	80.00
619	-34.44	83.15	80.00
620	-17.56	88.27	80.00
621	0.0	90.00	95.00
622	34.44	83.15	95.00
623	63.64	63.64	95.00
624	83.15	34.44	95.00
625	90.00	0.0	95.00
626	83.15	-34.44	95.00
627	63.64	-63.64	95.00
628	34.44	-83.15	95.00
629	0.0	-90.00	95.00
630	-34.44	-83.15	95.00
631	-63.64	-63.64	95.00
632	-83.15	-34.44	95.00
633	-90.00	0.0	95.00
634	-83.15	34.44	95.00
635	-63.64	63.64	95.00
636	-34.44	83.15	95.00
637	0.0	90.00	110.00
638	17.56	88.27	110.00
639	34.44	83.15	110.00
640	50.00	74.83	110.00
641	63.64	63.64	110.00
642	74.83	50.00	110.00
643	83.15	34.44	110.00
644	88.27	17.56	110.00
645	90.00	0.0	110.00
646	88.27	-17.56	110.00
647	83.15	-34.44	110.00
648	74.83	-50.00	110.00
649	63.64	-63.64	110.00
650	50.00	-74.83	110.00

651	34.44	-83.15	110.00
652	17.56	-88.27	110.00
653	0.0	-90.00	110.00
654	-17.56	-88.27	110.00
655	-34.44	-83.15	110.00
656	-50.00	-74.83	110.00
657	-63.64	-63.64	110.00
658	-74.83	-50.00	110.00
659	-83.15	-34.44	110.00
660	-88.27	-17.56	110.00
661	-90.00	0.0	110.00
662	-88.27	17.56	110.00
663	-83.15	34.44	110.00
664	-74.83	50.00	110.00
665	-63.64	63.64	110.00
666	-50.00	74.83	110.00
667	-34.44	83.15	110.00
668	-17.56	88.27	110.00
669	0.0	75.00	110.00
670	16.80	71.60	110.00
671	38.10	62.90	110.00
672	53.03	53.03	110.00
673	62.80	38.60	110.00
674	71.30	17.00	110.00
675	75.00	0.0	110.00
676	71.30	-17.00	110.00
677	62.80	-38.60	110.00
678	53.03	-53.03	110.00
679	38.10	-62.90	110.00
680	17.10	-71.80	110.00
681	0.0	-75.00	110.00
682	-16.90	-71.80	110.00
683	-38.50	-62.80	110.00
684	-53.03	-53.03	110.00
685	-62.70	-38.60	110.00
686	-71.30	-17.10	110.00
687	-75.00	0.0	110.00
688	-71.30	17.10	110.00
689	-62.90	38.30	110.00
690	-53.03	53.03	110.00
691	-38.60	62.80	110.00
692	-17.30	71.30	110.00
693	0.0	60.00	110.00
694	22.96	55.43	110.00
695	42.43	42.43	110.00
696	55.43	22.96	110.00
697	60.00	0.0	110.00
698	55.43	-22.96	110.00

699	42.43	-42.43	110.00
700	22.96	-55.43	110.00
701	0.0	-60.00	110.00
702	-22.96	-55.43	110.00
703	-42.43	-42.43	110.00
704	-55.43	-22.96	110.00
705	-60.00	0.0	110.00
706	-55.43	22.96	110.00
707	-42.43	42.43	110.00
708	-22.96	55.43	110.00
709	0.0	30.00	110.00
710	21.21	21.21	110.00
711	30.00	0.0	110.00
712	21.21	-21.21	110.00
713	0.0	-30.00	110.00
714	-21.21	-21.21	110.00
715	-30.00	0.0	110.00
716	-21.21	21.21	110.00
717	0.0	0.0	110.00

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C-----------------------------------------------------------------------
C        THIS PROGRAM DERIVES THE NAME VECTOR THROUGH THE USE OF THE
C        CONNECTIVITY MATRIX.
C-----------------------------------------------------------------------
C
C
      INTEGER*2 CONN,JA,MAME,NAME,JB
      DIMENSION CONN(128,15),JA(183),MAME(183,94),NAME(4615),JB(184)
    5 FORMAT(16I5)
    6 FORMAT(16I5)
   20 FORMAT(//2X,'ELEMENT',1X,'NODE 1',1X,'NODE 2',1X,'NODE 3',1X,'NODE
     1 4',1X,'NODE 5',1X,'NODE 6',1X,'NODE 7',1X,'NODE 8',1X,'NODE 9',1X
     1,'NODE10',1X,'NODE11',1X,'NODE12',1X,'NODE13',1X,'NODE14',1X,'NODE
     115')
   25 FORMAT(I6,15I7)
  205 FORMAT(25I5)
  204 FORMAT('0      JA(I) VECTOR')
  207 FORMAT('0      JB(I) VECTOR')
  208 FORMAT('0      NAME(JC) VECTOR')
  215 FORMAT(/2X,'JC=',I10)
C
C     THIS PROGRAM CALCULATES THE NAME ARRAY
C
      NUMNP=183
      NUMEL=128
      NELDOF=15
      DO 455 I=1,128
      READ(5,150)I,(CONN(I,J),J=1,15)
  150 FORMAT(16I5)
      WRITE(6,150)I,(CONN(I,J),J=1,15)
  455 CONTINUE
      DO 40 I=1,NUMNP
      JA(I)=1
   40 CONTINUE
      DO 60 I=1,NUMNP
      DO 50 J=1,93
      MAME(I,J)=0
   50 CONTINUE
   60 CONTINJE
      DO 65 I=1,NUMNP
      MAME(I,1)=I
   65 CONTINUE
      DO 100 I=1,NUMEL
      DO 90 J=1,NELDOF
      JJ=CONN(I,J)
      IF(JJ.GT.183) GO TO 90
      DO 80 K=1,NELDOF
      IF(K.EQ.J) GO TO 80
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      KK=CONN(I,K)
      IF(KK.GT.183) GO TO 80
      JA(JJ)=JA(JJ)+1
      JAA=JA(JJ)
      DO 70 L=2,JAA
      JJL=MAME(JJ,L)
      IF(JJL.EQ.KK) JA(JJ)=JA(JJ)-1
      IF(JJL.EQ.KK) GO TO 80
      IF(JJL.EQ.0) MAME(JJ,JAA)=KK
   70 CONTINUE
   80 CONTINUE
   90 CONTINUE
  100 CONTINUE
C
      JB(1)=1
      JC=0
      DO 200 I=1,183
      JN=JA(I)
      JB(I+1)=JB(I)+JA(I)
      JC=JC+JA(I)
  200 CONTINUE
      WRITE(6,215) JC
      DO 250 I=1,183
      JAA=JA(I)
      JBL=JB(I)
      DO 240 J=1,JAA
      JJ=JBL+J-1
      NAME(JJ)=MAME(I,J)
  240 CONTINUE
  250 CONTINUE
      WRITE(6,204)
      WRITE(6,205)(JA(I),I=1,183)
      WRITE(6,207)
      WRITE(6,205)(JB(I),I=1,183)
      WRITE(6,208)
      WRITE(6,205) (NAME(I),I=1,JC)
      STOP
      END
```

```
C
C------------------------------------------------------------------------
C     THIS IS THE MAIN PROGRAM WHICH CALCULATES THE BIGG AND BIGGG
C     SYSTEM MATRICES BY USING SUBROUTINES F_OWIE AND TANYA. THE SYSTEM
C     MATRICES ARE THEN PUT ON TAPE NUMBER NPS182.
C------------------------------------------------------------------------
C
      REAL*8 G,GG,BIGG,BIGGG
      REAL*8 WL,WS,S,CL1,CL2,CL3,FN,DL1,DL2,DL3,DS,DETJ                 M-010
      INTEGER*2 NAME,JA,JB,NNZ,JC,NUMNP,NUMEL,NELDOF,CONN               M-015
      COMMON/GTRY2/NAME(4615),JA(183),JB(183),NNZ,JC,NUMNP,NUMEL,NELDOF,
     1CONN(128,15)
      COMMON/GTRY1/X(505),P(505),Z(505)
      COMMON/XOCAL/XX(15),YY(15),ZZ(15),PPSI(15)                        M-060
      COMMON/GMAT/G(15,15)
      COMMON/GGMAT/GG(15,15)
      COMMON/SYSMT1/BIGG(4615)
      COMMON/SYSMT2/BIGGG(4615)
      COMMON/GAUSS/WL(7),WS(5),S(5),CL1(7),CL2(7),CL3(7),FN(15,21),DL1(1
     15,21),DL2(15,21),DL3(15,21),DS(15,21),DETJ(21)                    M-100
  150 FORMAT(16I5)
  160 FORMAT(25I5)
  211 FORMAT(8E16.8)
  609 FORMAT(12F10.4)
  702 FORMAT(20I6)
  800 FORMAT(10F8.2)
  952 FORMAT(20I4)
      NNZ=183
      NUMEL=128
      NUMNP=505
      JC=4615
      NELDOF=15
      DO 455 I=1,NUMEL                                                  M-120
      READ(5,150)I,(CONN(I,J),J=1,NELDOF)                               M-125
  455 CONTINUE                                                          M-130
      READ(5,952)(JA(I),I=1,NNZ)
      READ(5,952)(JB(I),I=1,NNZ)
      READ(5,952)(NAME(I),I=1,JC)                                       M-160
      READ(5,800)(X(I),I=1,NUMNP)                                       M-140
      READ(5,800)(P(I),I=1,NUMNP)                                       M-145
      READ(5,800)(Z(I),I=1,NUMNP)
      DO 465 I=1,JC                                                     M-355
      BIGG(I)=0.0                                                       M-360
      BIGGG(I)=0.0                                                      M-365
  465 CONTINUE                                                          M-370
      CALL TANYA                                                        M-405
      CALL FLOWIE                                                       M-415
      STOP                                                              M-615
```

```
      END                                                                M-620
C
C-----------------------------------------------------------------------
C     THIS SUBROUTINE EVALUATES THE GG ELEMENT MATRIX USING 20 POINTS OF
C     INTEGRATION. THE GG ELEMENT MATRIX IS THEN INSERTED INTO THE BIGGG
C     SYSTEM MATRIX.
C-----------------------------------------------------------------------
C
      SUBROUTINE TANYA
      REAL*8 GG,BIGGG
      REAL*8 WL,WS,S,CL1,CL2,CL3,FN,DL1,DL2,DL3,DS,DETJ                  T-015
      INTEGER*2 NAME,JA,JB,NNZ,JC,NJMNP,NUMEL,NELDOF,CONN
      COMMON/GTRY2/NAME(8199),JA(299),JB(299),NNZ,JC,NUMNP,NUMEL,NELDOF,
     1CONN(192,15)
      COMMON/GTRY1/X(717),P(717),Z(717)
      COMMON/XOCAL/KX(15),YY(15),ZZ(15),PPSI(15)
      COMMON/GGMAT/GG(15,15)
      COMMON/SYSMT2/BIGGG(8199)
      COMMON/GAUSS/WL(7),WS(5),S(5),CL1(7),CL2(7),CL3(7),FN(15,21),DL1(1
     15,21),DL2(15,21),DL3(15,21),DS(15,21),DETJ(21)
      WS(1)=0.236926885056189
      WS(2)=0.478628670499366
      WS(3)=0.568888888888889
      WS(4)=0.478628670499366
      WS(5)=0.236926885056189
      S(1)=0.906179845938664
      S(2)=0.538469310105683
      S(3)=0.0
      S(4)=-.538469310105683
      S(5)=-.906179845938664
      WL(1)=-0.281250
      WL(2)=0.260416666666667
      WL(3)=0.260416666666667
      WL(4)=0.260416666666667
      CL1(1)=0.333333333333333
      CL1(2)=0.60
      CL1(3)=0.20
      CL1(4)=0.20
      CL2(1)=0.333333333333333
      CL2(2)=0.20
      CL2(3)=0.60
      CL2(4)=0.20
      CL3(1)=0.333333333333333
      CL3(2)=0.20
      CL3(3)=0.20
      CL3(4)=0.60
      DO 80 K=1,5
      DO 90 M=1,4
```

```
      IF(K.EQ.1) GO TO 10
      IF(K.EQ.2) GO TO 20
      IF(K.EQ.3) GO TO 30
      IF(K.EQ.4) GO TO 40
      IF(K.EQ.5) GO TO 50
   10 N=M
      GO TO 60
   20 N=M+4
      GO TO 60
   30 N=M+8
      GO TO 60
   40 N=M+12
      GO TO 60
   50 N=M+16
   60 CONTINUE
      FN(1,N)= 0.50*CL1(M)*((2.0*CL1(M)-1.0)*(1.0+S(K))-(1.0-S(K)**2))
      FN(2,N)=  2.0*CL1(M)*CL2(M)*(1.0+S(K))
      FN(3,N)= 0.50*CL2(M)*((2.0*CL2(M)-1.0)*(1.0+S(K))-(1.0-S(K)**2))
      FN(4,N)=  2.0*CL2(M)*CL3(M)*(1.0+S(K))
      FN(5,N)= 0.50*CL3(M)*((2.0*CL3(M)-1.0)*(1.0+S(K))-(1.0-S(K)**2))
      FN(6,N)=  2.0*CL1(M)*CL3(M)*(1.0+S(K))
      FN(7,N)= CL1(M)*(1.0-S(K)**2)
      FN(8,N)= CL2(M)*(1.0-S(K)**2)
      FN(9,N)= CL3(M)*(1.0-S(K)**2)
      FN(10,N)=0.50*CL1(M)*((2.0*CL1(M)-1.0)*(1.0-S(K))-(1.0-S(K)**2))
      FN(11,N)= 2.0*CL1(M)*CL2(M)*(1.0-S(K))
      FN(12,N)=0.50*CL2(M)*((2.0*CL2(M)-1.0)*(1.0-S(K))-(1.0-S(K)**2))
      FN(13,N)= 2.0*CL2(M)*CL3(M)*(1.0-S(K))
      FN(14,N)=0.50*CL3(M)*((2.0*CL3(M)-1.0)*(1.0-S(K))-(1.0-S(K)**2))
      FN(15,N)= 2.0*CL1(M)*CL3(M)*(1.0-S(K))
      DL1(1,N)= 0.50*((1.0+S(K))*(4.0*CL1(M)-1.0)-(1.0-S(K)**2))
      DL1(2,N)=  2.0*CL2(M)*(1.0+S(K))
      DL1(3,N)=0.0
      DL1(4,N)=0.0
      DL1(5,N)=0.0
      DL1(6,N)= 2.0*CL3(M)*(1.0+S(K))
      DL1(7,N)= (1.0-S(K)**2)
      DL1(8,N)=0.0
      DL1(9,N)=0.0
      DL1(10,N)=0.50*((1.0-S(K))*(4.0*CL1(M)-1.0)-(1.0-S(K)**2))
      DL1(11,N)= 2.0*CL2(M)*(1.0-S(K))
      DL1(12,N)=0.0
      DL1(13,N)=0.0
      DL1(14,N)=0.0
      DL1(15,N)= 2.0*CL3(M)*(1.0-S(K))
      DL2(1,N)=0.0
      DL2(2,N)=  2.0*CL1(M)*(1.0+S(K))
      DL2(3,N)= 0.50*((1.0+S(K))*(4.0*CL2(M)-1.0)-(1.0-S(K)**2))
```

```
      DL2(4,N)=  2.0*CL3(M)*(1.0+S(K))
      DL2(5,N)=0.0
      DL2(6,N)=0.0
      DL2(7,N)=0.0
      DL2(8,N)= (1.0-S(K)**2)
      DL2(9,N)=0.0
      DL2(10,N)=0.0
      DL2(11,N)= 2.0*CL1(M)*(1.0-S(K))
      DL2(12,N)=0.50*((1.0-S(K))*(4.0*CL2(M)-1.0)-(1.0-S(K)**2))
      DL2(13,N)= 2.0*CL3(M)*(1.0-S(K))
      DL2(14,N)=0.0
      DL2(15,N)=0.0
      DL3(1,N)=0.0
      DL3(2,N)=0.0
      DL3(3,N)=0.0
      DL3(4,N)=  2.0*CL2(M)*(1.0+S(K))
      DL3(5,N)= 0.50*((1.0+S(K))*(4.0*CL3(M)-1.0)-(1.0-S(K)**2))
      DL3(6,N)=  2.0*CL1(M)*(1.0+S(K))
      DL3(7,N)=0.0
      DL3(8,N)=0.0
      DL3(9,N)= (1.0-S(K)**2)
      DL3(10,N)=0.0
      DL3(11,N)=0.0
      DL3(12,N)=0.0
      DL3(13,N)= 2.0*CL2(M)*(1.0-S(K))
      DL3(14,N)=0.50*((1.0-S(K))*(4.0*CL3(M)-1.0)-(1.0-S(K)**2))
      DL3(15,N)= 2.0*CL1(M)*(1.0-S(K))
      DS(1,N)=0.50*(2.0*CL1(M)**2-CL1(M))+CL1(M)*S(K)
      DS(2,N)=  2.0*CL1(M)*CL2(M)
      DS(3,N)=0.50*(2.0*CL2(M)**2-CL2(M))+CL2(M)*S(K)
      DS(4,N)=  2.0*CL2(M)*CL3(M)
      DS(5,N)=0.50*(2.0*CL3(M)**2-CL3(M))+CL3(M)*S(K)
      DS(6,N)=  2.0*CL1(M)*CL3(M)
      DS(7,N)= -2.0*CL1(M)*S(K)
      DS(8,N)= -2.0*CL2(M)*S(K)
      DS(9,N)= -2.0*CL3(M)*S(K)
      DS(10,N)=-.50*(2.*CL1(M)**2-CL1(M))+CL1(M)*S(K)
      DS(11,N)= -2.0*CL1(M)*CL2(M)
      DS(12,N)=-.50*(2.*CL2(M)**2-CL2(M))+CL2(M)*S(K)
      DS(13,N)= -2.0*CL2(M)*CL3(M)
      DS(14,N)=-.50*(2.*CL3(M)**2-CL3(M))+CL3(M)*S(K)
      DS(15,N)= -2.0*CL1(M)*CL3(M)
   90 CONTINUE
   80 CONTINUE
      DO 510 L=1,NUMEL
      DO 475 I=1,NELDOF
      DO 470 J=1,NELDOF
      GG(I,J)=0.0
```

```
470 CONTINUE
475 CONTINUE
    DO 480 JJ=1,NELDOF
    NN=CONN(L,JJ)
    XX(JJ)=X(NN)
    YY(JJ)=P(NN)
    ZZ(JJ)=Z(NN)
480 CONTINUE
    A5=2.0/(ZZ(1)-ZZ(15))
    DETJJ=0.0
    DO 200 K=1,5
    DETJ1=0.0
    DO 210 M=1,4
    IF(K.EQ.1) GO TO 11
    IF(K.EQ.2) GO TO 21
    IF(K.EQ.3) GO TO 31
    IF(K.EQ.4) GO TO 41
    IF(K.EQ.5) GO TO 51
 11 N=M
    GO TO 61
 21 N=M+4
    GO TO 61
 31 N=M+8
    GO TO 61
 41 N=M+12
    GO TO 61
 51 N=M+16
 61 CONTINUE
    T1=DL1(1,N)*XX(1)+DL1(2,N)*XX(2)-DL3(4,N)*XX(4)-DL3(5,N)*XX(5)
    T2=DL1(7,N)*XX(7)-DL3(9,N)*XX(9)+DL1(10,N)*XX(10)+DL1(11,N)*XX(11)
    T3=(DL1(6,N)-DL3(6,N))*XX(6)-DL3(13,N)*XX(13)-DL3(14,N)*XX(14)
    T4=(DL1(15,N)-DL3(15,N))*XX(15)
    DJ11=T1+T2+T3+T4
    D1=DL1(1,N)*YY(1)+DL1(2,N)*YY(2)-DL3(4,N)*YY(4)-DL3(5,N)*YY(5)
    D2=DL1(7,N)*YY(7)-DL3(9,N)*YY(9)+DL1(10,N)*YY(10)+DL1(11,N)*YY(11)
    D3=(DL1(6,N)-DL3(6,N))*YY(6)-DL3(13,N)*YY(13)-DL3(14,N)*YY(14)
    D4=(DL1(15,N)-DL3(15,N))*YY(15)
    DJ12=D1+D2+D3+D4
    T5=DL2(2,N)*XX(2)+DL2(3,N)*XX(3)-DL3(5,N)*XX(5)-DL3(6,N)*XX(6)
    T6=DL2(8,N)*XX(8)-DL3(9,N)*XX(9)+(DL2(4,N)-DL3(4,N))*XX(4)
    T7=DL2(11,N)*XX(11)+DL2(12,N)*XX(12)+(DL2(13,N)-DL3(13,N))*XX(13)
    T8=-DL3(14,N)*XX(14)-DL3(15,N)*XX(15)
    DJ21=T5+T6+T7+T8
    D5=DL2(2,N)*YY(2)+DL2(3,N)*YY(3)-DL3(5,N)*YY(5)-DL3(6,N)*YY(6)
    D6=DL2(8,N)*YY(8)-DL3(9,N)*YY(9)+(DL2(4,N)-DL3(4,N))*YY(4)
    D7=DL2(11,N)*YY(11)+DL2(12,N)*YY(12)+(DL2(13,N)-DL3(13,N))*YY(13)
    D8=-DL3(14,N)*YY(14)-DL3(15,N)*YY(15)
    DJ22=D5+D6+D7+D8
```

```
      DETJ(N)=DJ11*DJ22-DJ12*DJ21
      DETJ1=DETJ1+DETJ(N)*WL(M)
  210 CONTINUE
      DETJJ=DETJJ+WS(K)*DETJ1
  200 CONTINUE
      DO 600  J=1,15
      DO 610  I=1,15
      FGG=0.0
      DO 660  K=1,5
      FFG=0.0
      DO 670  M=1,4
      IF(K.EQ.1)  GO  TO  12
      IF(K.EQ.2)  GO  TO  22
      IF(K.EQ.3)  GO  TO  32
      IF(K.EQ.4)  GO  TO  42
      IF(K.EQ.5)  GO  TO  52
   12 N=M
      GO  TO  62
   22 N=M+4
      GO  TO  62
   32 N=M+8
      GO  TO  62
   42 N=M+12
      GO  TO  62
   52 N=M+16
   62 CONTINUE
      DXDL5=DL1(1,N)*XX(1)+(DL1(2,N)-DL2(2,N))*XX(2)-DL2(3,N)*XX(3)
      DXDL6=-(DL2(4,N)+DL3(4,N))*XX(4)-DL3(5,N)*XX(5)
      DXDL7=(DL1(6,N)-DL3(6,N))*XX(6)+DL1(7,N)*XX(7)-DL2(8,N)*XX(8)
      DXDL8=-DL3(9,N)*XX(9)+DL1(10,N)*XX(10)+(DL1(11,N)-DL2(11,N))*XX(11
     1)
      DXDL9=-DL2(12,N)*XX(12)-(DL2(13,N)+DL3(13,N))*XX(13)-DL3(14,N)*XX(
     114)+(DL1(15,N)-DL3(15,N))*XX(15)
      DXDL1=DXDL5+DXDL6+DXDL7+DXDL8+DXDL9
      DYDL5=DL1(1,N)*YY(1)+(DL1(2,N)-DL2(2,N))*YY(2)-DL2(3,N)*YY(3)
      DYDL6=-(DL2(4,N)+DL3(4,N))*YY(4)-DL3(5,N)*YY(5)
      DYDL7=(DL1(6,N)-DL3(6,N))*YY(6)+DL1(7,N)*YY(7)-DL2(8,N)*YY(8)
      DYDL8=-DL3(9,N)*YY(9)+DL1(10,N)*YY(10)+(DL1(11,N)-DL2(11,N))*YY(11
     1)
      DYDL9=-DL2(12,N)*YY(12)-(DL2(13,N)+DL3(13,N))*YY(13)-DL3(14,N)*YY(
     114)+(DL1(15,N)-DL3(15,N))*YY(15)
      DYDL1=DYDL5+DYDL6+DYDL7+DYDL8+DYDL9
      DXDL55=-DL1(1,N)*XX(1)+(DL2(2,N)-DL1(2,N))*XX(2)+DL2(3,N)*XX(3)
      DXDL66=(DL2(4,N)-DL3(4,N))*XX(4)-DL3(5,N)*XX(5)-DL1(7,N)*XX(7)
      DXDL77=-(DL1(6,N)+DL3(6,N))*XX(6)+DL2(8,N)*XX(8)-DL3(9,N)*XX(9)
      DXDL88=-DL1(10,N)*XX(10)+(DL2(11,N)-DL1(11,N))*XX(11)
      DXDL99=DL2(12,N)*XX(12)+(DL2(13,N)-DL3(13,N))*XX(13)-DL3(14,N)*XX(
     114)-(DL1(15,N)+DL3(15,N))*XX(15)
```

```
      DXDL2=DXDL55+DXDL66+DXDL77+DXDL88+DXDL99
      DYDL55=-DL1(1,N)*YY(1)+(DL2(2,N)-DL1(2,N))*YY(2)+DL2(3,N)*YY(3)
      DYDL66=(DL2(4,N)-DL3(4,N))*YY(4)-DL3(5,N)*YY(5)-DL1(7,N)*YY(7)
      DYDL77=-(DL1(6,N)+DL3(6,N))*YY(6)+DL2(8,N)*YY(8)-DL3(9,N)*YY(9)
      DYDL88=-DL1(10,N)*YY(10)+(DL2(11,N)-DL1(11,N))*YY(11)
      DYDL99=DL2(12,N)*YY(12)+(DL2(13,N)-DL3(13,N))*YY(13)-DL3(14,N)*YY(
     114)-(DL1(15,N)+DL3(15,N))*YY(15)
      DYDL2=DYDL55+DYDL66+DYDL77+DYDL88+DYDL99
      DL1DY=1.0/DYDL1
      DL2DX=1.0/DXDL2
      DL1DX=1.0/DXDL1
      DL2DY=1.0/DYDL2
      BB11=(DL1DX**2)+(DL1DY**2)
      BB12=DL1DX*DL2DX+DL1DY*DL2DY
      BB22=(DL2DX**2)+(DL2DY**2)
      H1=BB11*((DL1(J,N)-DL3(J,N))*(DL1(I,N)-DL3(I,N)))
      H2=BB12*((DL2(J,N)-DL3(J,N))*(DL1(I,N)-DL3(I,N)))
      H3=BB12*((DL1(J,N)-DL3(J,N))*(DL2(I,N)-DL3(I,N)))
      H4=BB22*((DL2(J,N)-DL3(J,N))*(DL2(I,N)-DL3(I,N)))
      H5=(A5**2)*DS(J,N)*DS(I,N)
      H6=(H1+H2+H3+H4+H5)*DETJ(N)
      FFG=FFG+WL(M)*H6
  670 CONTINUE
      FGG=WS(K)*FFG+FGG
  660 CONTINUE
      GG(J,I)=FGG/A5
  610 CONTINUE
  600 CONTINUE
C     *** INSERT ELEMENT MATRIX INTO SYSTEM MATRIX *****
      DO 505 K=1,NELDOF
      KK=CONN(L,K)
      IF(KK.GT.NNZ) GO TO 505
      KKK=JA(KK)
      LLL=JB(KK)-1
      DO 500 I=1,NELDOF
      II=CONN(L,I)
      IF(II.GT.NNZ) GO TO 500
      DO 490 M=1,KKK
      MM=LLL+M
      KKM=NAME(MM)
      IF(II.EQ.KKM) GO TO 495
  490 CONTINUE
  495 CONTINUE
      BIGGG(MM)=BIGGG(MM)+GG(K,I)
  500 CONTINUE
  505 CONTINUE
  510 CONTINUE
      RETURN
```

```
      END
C
C-----------------------------------------------------------------------
C     THIS SUBROUTINE EVALJATES THE G ELEMENT MATRIX USING  21 POINTS OF
C     INTEGRATION. THE G ELEMENT MATRIX IS THEN INSERTED INTO THE BIGG
C     SYSTEM MATRIX.
C-----------------------------------------------------------------------
C
      SUBROUTINE FLOWIE
      REAL*8 G,BIGG
      REAL*8 WL,WS,S,CL1,CL2,CL3,FN,DL1,DL2,DL3,DS,DETJ
      INTEGER*2 NAME,JA,JB,NNZ,JC,NJMNP,NUMEL,NELDOF,CONN
      COMMON/GTRY2/NAME(8199),JA(299),JB(299),NNZ,JC,NUMNP,NUMEL,NELDOF,
     1CONN(192,15)
      COMMON/GTRY1/X(717),P(717),Z(717)
      COMMON/XOCAL/XX(15),YY(15),ZZ(15),PPSI(15)
      COMMON/GMAT/G(15,15)
      COMMON/SYSMT1/BIGG(8199)
      COMMON/GAUSS/WL(7),WS(5),S(5),CL1(7),CL2(7),CL3(7),FN(15,21),DL1(1
     15,21),DL2(15,21),DL3(15,21),DS(15,21),DETJ(21)
      WS(1)=0.555555555555556
      WS(2)=0.888888888888889
      WS(3)=0.555555555555556
      S(1)=0.774596669241483
      S(2)=0.0
      S(3)=-.774596669241483
      WL(1)=0.112500
      WL(2)=0.066197007500
      WL(3)=0.066197007500
      WL(4)=0.066197007500
      WL(5)=0.0629695900
      WL(6)=0.0629695900
      WL(7)=0.0629695900
      CL1(1)=0.333333333333333
      CL1(2)=0.059615870
      CL1(3)=0.470142060
      CL1(4)=0.470142060
      CL1(5)=0.797426990
      CL1(6)=0.101286510
      CL1(7)=0.101286510
      CL2(1)=0.333333333333333
      CL2(2)=0.470142060
      CL2(3)=0.059615870
      CL2(4)=0.470142060
      CL2(5)=0.101286510
      CL2(6)=0.797426990
      CL2(7)=0.101286510
      CL3(1)=0.333333333333333
```

```
      CL3(2)=0.470142060
      CL3(3)=0.470142060
      CL3(4)=0.059615870
      CL3(5)=0.101286510
      CL3(6)=0.101286510
      CL3(7)=0.797426990
      DO 10 K=1,3
      DO 20 M=1,7
C     THE 3 X 7 MATRICES ARE BEING CONVERTED TO 1 X 21 VECTORS
      IF(K-2)30,40,50
   30 N=M
      GO TO 60
   40 N=M+7
      GO TO 60
   50 N=M+14
   60 CONTINUE
      FN(1,N)= 0.50*CL1(M)*((2.0*CL1(M)-1.0)*(1.0+S(K))-(1.0-S(K)**2))
      FN(2,N)=  2.0*CL1(M)*CL2(M)*(1.0+S(K))
      FN(3,N)= 0.50*CL2(M)*((2.0*CL2(M)-1.0)*(1.0+S(K))-(1.0-S(K)**2))
      FN(4,N)=  2.0*CL2(M)*CL3(M)*(1.0+S(K))
      FN(5,N)= 0.50*CL3(M)*((2.0*CL3(M)-1.0)*(1.0+S(K))-(1.0-S(K)**2))
      FN(6,N)=  2.0*CL1(M)*CL3(M)*(1.0+S(K))
      FN(7,N)= CL1(M)*(1.0-S(K)**2)
      FN(8,N)= CL2(M)*(1.0-S(K)**2)
      FN(9,N)= CL3(M)*(1.0-S(K)**2)
      FN(10,N)=0.50*CL1(M)*((2.0*CL1(M)-1.0)*(1.0-S(K))-(1.0-S(K)**2))
      FN(11,N)= 2.0*CL1(M)*CL2(M)*(1.0-S(K))
      FN(12,N)=0.50*CL2(M)*((2.0*CL2(M)-1.0)*(1.0-S(K))-(1.0-S(K)**2))
      FN(13,N)= 2.0*CL2(M)*CL3(M)*(1.0-S(K))
      FN(14,N)=0.50*CL3(M)*((2.0*CL3(M)-1.0)*(1.0-S(K))-(1.0-S(K)**2))
      FN(15,N)= 2.0*CL1(M)*CL3(M)*(1.0-S(K))
      DL1(1,N)= 0.50*((1.0+S(K))*(4.0*CL1(M)-1.0)-(1.0-S(K)**2))
      DL1(2,N)=  2.0*CL2(M)*(1.0+S(K))
      DL1(3,N)=0.0
      DL1(4,N)=0.0
      DL1(5,N)=0.0
      DL1(6,N)= 2.0*CL3(M)*(1.0+S(K))
      DL1(7,N)= (1.0-S(K)**2)
      DL1(8,N)=0.0
      DL1(9,N)=0.0
      DL1(10,N)=0.50*((1.0-S(K))*(4.0*CL1(M)-1.0)-(1.0-S(K)**2))
      DL1(11,N)= 2.0*CL2(M)*(1.0-S(K))
      DL1(12,N)=0.0
      DL1(13,N)=0.0
      DL1(14,N)=0.0
      DL1(15,N)= 2.0*CL3(M)*(1.0-S(K))
      DL2(1,N)=0.0
      DL2(2,N)=  2.0*CL1(M)*(1.0+S(K))
```

```
      DL2(3,N)= 0.50*((1.0+S(K))*(4.0*CL2(M)-1.0)-(1.0-S(K)**2))
      DL2(4,N)=  2.0*CL3(M)*(1.0+S(K))
      DL2(5,N)=0.0
      DL2(6,N)=0.0
      DL2(7,N)=0.0
      DL2(8,N)= (1.0-S(K)**2)
      DL2(9,N)=0.0
      DL2(10,N)=0.0
      DL2(11,N)= 2.0*CL1(M)*(1.0-S(K))
      DL2(12,N)=0.50*((1.0-S(K))*(4.0*CL2(M)-1.0)-(1.0-S(K)**2))
      DL2(13,N)= 2.0*CL3(M)*(1.0-S(K))
      DL2(14,N)=0.0
      DL2(15,N)=0.0
      DL3(1,N)=0.0
      DL3(2,N)=0.0
      DL3(3,N)=0.0
      DL3(4,N)=  2.0*CL2(M)*(1.0+S(K))
      DL3(5,N)= 0.50*((1.0+S(K))*(4.0*CL3(M)-1.0)-(1.0-S(K)**2))
      DL3(6,N)=  2.0*CL1(M)*(1.0+S(K))
      DL3(7,N)=0.0
      DL3(8,N)=0.0
      DL3(9,N)= (1.0-S(K)**2)
      DL3(10,N)=0.0
      DL3(11,N)=0.0
      DL3(12,N)=0.0
      DL3(13,N)= 2.0*CL2(M)*(1.0-S(K))
      DL3(14,N)=0.50*((1.0-S(K))*(4.0*CL3(M)-1.0)-(1.0-S(K)**2))
      DL3(15,N)= 2.0*CL1(M)*(1.0-S(K))
      DS(1,N)=0.50*(2.0*CL1(M)**2-CL1(M))+CL1(M)*S(K)
      DS(2,N)=  2.0*CL1(M)*CL2(M)
      DS(3,N)=0.50*(2.0*CL2(M)**2-CL2(M))+CL2(M)*S(K)
      DS(4,N)=  2.0*CL2(M)*CL3(M)
      DS(5,N)=0.50*(2.0*CL3(M)**2-CL3(M))+CL3(M)*S(K)
      DS(6,N)=  2.0*CL1(M)*CL3(M)
      DS(7,N)= -2.0*CL1(M)*S(K)
      DS(8,N)= -2.0*CL2(M)*S(K)
      DS(9,N)= -2.0*CL3(M)*S(K)
      DS(10,N)=-.50*(2.*CL1(M)**2-CL1(M))+CL1(M)*S(K)
      DS(11,N)= -2.0*CL1(M)*CL2(M)
      DS(12,N)=-.50*(2.*CL2(M)**2-CL2(M))+CL2(M)*S(K)
      DS(13,N)= -2.0*CL2(M)*CL3(M)
      DS(14,N)=-.50*(2.*CL3(M)**2-CL3(M))+CL3(M)*S(K)
      DS(15,N)= -2.0*CL1(M)*CL3(M)
   20 CONTINUE
   10 CONTINUE
      DO 510 L=1,NUMEL
      DO 475 I=1,NELDOF
      DO 470 J=1,NELDOF
```

```
      G(I,J)=0.0
  470 CONTINUE
  475 CONTINUE
      DO 480 JJ=1,NELDOF
      NN=CONN(L,JJ)
      XX(JJ)=X(NN)
      YY(JJ)=P(NN)
      ZZ(JJ)=Z(NN)
  480 CONTINUE
      A5=2.0/(ZZ(1)-ZZ(15))
      DETJJ=0.0
      DO 200 K=1,3
      DETJ1=0.0
      DO 210 M=1,7
      IF(K-2)31,41,51
   31 N=M
      GO TO 61
   41 N=M+7
      GO TO 61
   51 N=M+14
   61 CONTINUE
      T1=DL1(1,N)*XX(1)+DL1(2,N)*XX(2)-DL3(4,N)*XX(4)-DL3(5,N)*XX(5)
      T2=DL1(7,N)*XX(7)-DL3(9,N)*XX(9)+DL1(10,N)*XX(10)+DL1(11,N)*XX(11)
      T3=(DL1(6,N)-DL3(6,N))*XX(6)-DL3(13,N)*XX(13)-DL3(14,N)*XX(14)
      T4=(DL1(15,N)-DL3(15,N))*XX(15)
      DJ11=T1+T2+T3+T4
      D1=DL1(1,N)*YY(1)+DL1(2,N)*YY(2)-DL3(4,N)*YY(4)-DL3(5,N)*YY(5)
      D2=DL1(7,N)*YY(7)-DL3(9,N)*YY(9)+DL1(10,N)*YY(10)+DL1(11,N)*YY(11)
      D3=(DL1(6,N)-DL3(6,N))*YY(6)-DL3(13,N)*YY(13)-DL3(14,N)*YY(14)
      D4=(DL1(15,N)-DL3(15,N))*YY(15)
      DJ12=D1+D2+D3+D4
      T5=DL2(2,N)*XX(2)+DL2(3,N)*XX(3)-DL3(5,N)*XX(5)-DL3(6,N)*XX(6)
      T6=DL2(8,N)*XX(8)-DL3(9,N)*XX(9)+(DL2(4,N)-DL3(4,N))*XX(4)
      T7=DL2(11,N)*XX(11)+DL2(12,N)*XX(12)+(DL2(13,N)-DL3(13,N))*XX(13)
      T8=-DL3(14,N)*XX(14)-DL3(15,N)*XX(15)
      DJ21=T5+T6+T7+T8
      D5=DL2(2,N)*YY(2)+DL2(3,N)*YY(3)-DL3(5,N)*YY(5)-DL3(6,N)*YY(6)
      D6=DL2(8,N)*YY(8)-DL3(9,N)*YY(9)+(DL2(4,N)-DL3(4,N))*YY(4)
      D7=DL2(11,N)*YY(11)+DL2(12,N)*YY(12)+(DL2(13,N)-DL3(13,N))*YY(13)
      D8=-DL3(14,N)*YY(14)-DL3(15,N)*YY(15)
      DJ22=D5+D6+D7+D8
      DETJ(N)=DJ11*DJ22-DJ12*DJ21
      DETJ1=DETJ(N)*WL(M)+DETJ1
  210 CONTINUE
      DETJJ=DETJJ+WS(K)*DETJ1
  200 CONTINUE
      DO 600 J=1,15
      DO 610 I=1,15
```

```
      FG=0.0
      DO 660  K=1,3
      F=0.0
      DO 670  M=1,7
      IF(K-2)620,630,640
  620 N=M
      GO TO 650
  630 N=M+7
      GO TO 650
  640 N=M+14
  650 CONTINUE
      F=F+WL(M)*FN(J,N)*FN(I,N)*DETJ(N)
  670 CONTINUE
      FG=FG+WS(K)*F
  660 CONTINUE
      G(J,I)=FG/A5
  610 CONTINUE
  600 CONTINUE
C     *** INSERT ELEMENT MATRIX INTO SYSTEM MATRIX *****
      DO 505  K=1,NELDOF
      KK=CONN(L,K)
      IF(KK.GT.NNZ) GO TO 505
      KKK=JA(KK)
      LLL=JB(KK)-1
      DO 500  I=1,NELDOF
      II=CONN(L,I)
      IF(II.GT.NNZ) GO TO 500
      DO 490  M=1,KK<
      MM=LLL+M
      KKM=NAME(MM)
      IF(II.EQ.KKM) GO TO 495
  490 CONTINUE
  495 CONTINUE
      BIGG(MM)=BIGG(MM)+G(K,I)
  500 CONTINUE
  505 CONTINUE
  510 CONTINUE
      RETURN
      END
```

```
C
C-----------------------------------------------------------------------
C     THIS IS THE MAIN PROGRAM FOR THE SOLUTION OF THE FIELD EQUATIONS.
C     THE DATA FROM THE TAPE IS READ OUT, VJDAL CONSTANTS ARE EVALUATED,
C     THE INTEGRATION PACKAGE IS INITIATED,ETC.
C-----------------------------------------------------------------------
C
C
C
      REAL*8 SIGF,SIGA,C1,C2,C4,C5
      REAL*8 GGG,BIGG,BIGGG,BIGH
      REAL*8 WL,WS,S,CL1,CL2,CL3,FN,DL1,DL2,DL3,DS,DETJ                 M-010
      INTEGER*2 NAME,JA,JB,NNZ,JC,NJMNP,NUMEL,NELDOF,CONN               M-015
      COMMON/GTRY2/NAME(4615),JA(183),JB(184),NNZ,JC,NUMNP,NUMEL,NELDOF, M-020
     1CONN(128,15)                                                      M-025
      COMMON/GTRY1/X(505),P(505),Z(505)                                 M-055
      COMMON/XOCAL/XX(15),YY(15),ZZ(15),PPSI(15)                        M-060
      DIMENSION W(7J00),Y(7,183)
      COMMON/DELAY/SUM(183),PSI(93)
      COMMON/TEMP/C3(183),C6(15)
      DIMENSION SIGF(93),SIGA(183)
      COMMON/COEF/C1(183),C2(183),C4(183),C5(183)
      COMMJN/GGGMAT/GGG(15,15)                                          M-075
      COMMON/SYSMT1/BIGG(4615)                                          M-080
      COMM3N/SYSMT2/BIGGG(4615)                                         M-085
      COMMON/SYSMT3/BIGH(4615)
      COMMON/GAUSS/WL(7),WS(5),S(5),CL1(7),CL2(7),CL3(7),FN(15,21),DL1(1 M-095
     15,21),DL2(15,21),DL3(15,21),DS(15,21),DETJ(21)                    M-100
      NNZ=183
      NUMEL=128
      NUMNP=505
      NELDOF=15
      JC=4615                                                           M-105
      FMAX=1.0E10
      DO 771 I=1,NUMEL
      READ(4) (CONN(I,J),J=1,NELDOF)
  771 CONTINUE
      READ(4) (JA(I),I=1,NNZ)
      READ(4) (JB(I),I=1,NNZ)
      READ(4) (NAME(I),I=1,JC)
      READ(4) (X(I),I=1,NUMNP)
      READ(4) (P(I),I=1,NUMNP)
      READ(4) (Z(I),I=1,NUMNP)
      READ(4) (BIGG(I),I=1,JC)
      READ(4) (BIGGG(I),I=1,JC)
      DO 772 I=1,3
      READ(4) WS(I),S(I)
  772 CONTINUE
```

```
     DO 773 I=1,7
     READ(4) WL(I),CL1(I),CL2(I),CL3(I)
 773 CONTINUE
     DO 781 I=1,NELDOF
     DO 782 J=1,21
     READ(4) FN(I,J),DL1(I,J),DL2(I,J),DL3(I,J),DS(I,J)
 782 CONTINUE
 781 CONTINUE
     DO 460 I=1,NNZ                                                          M-520
     Y(1,I)=FMAX
     Y(2,I)=0.0                                                              M-535
     SUM(I)=0.0                                                              M-545
     IF(I.GT.93) GJ TO 460                                                   M-540
     PSI(I)=FMAX
 460 CONTINUE                                                                M-550
     WRITE(6,3)
   3 FORMAT('      SIGF(I)=0.0057450,   SIGA(I)=0.014010,0.0080   ')
     V=4.800E07
     Q=0.4349710
     BETA=0.006420
     B=-0.00400
     DO 445 I=1,NNZ
     IF(I.GT.93) GJ TO 420
     D=0.9130
     SIGF(I)=0.0057450
     SIGA(I)=0.014010
     ZNU=2.54
     C1(I)=V*D
     C2(I)=V*SIGA(I)-V*(1.0-BETA)*SIGF(I)*ZNU
     C3(I)=(2.0179325E-13)*SIGF(I)
     C4(I)=-V*(1.0-BETA)*SIGA(I)*B
     C5(I)=-V*BETA*Q*ZNU*SIGF(I)
     GO TO 445
 420 D=1.200
     SIGA(I)=0.00800
     C1(I)=V*D
     C2(I)=V*SIGA(I)
     C3(I)=0.0
     C4(I)=0.0
     C5(I)=0.0
 445 CONTINUE
     TEND=0.0010
     NY=NNZ                                                                  M-565
     NL=0                                                                    M-570
     H=1.0E-19                                                               M-575
     T=0.0                                                                   M-580
     HMIN=1.0E-20                                                            M-585
     HMAX=0.10                                                               M-590
```

```
      JSKF=0                                                               M-595
      CALL SDESOL(Y,YL,T,TEND,NY,NL,NY,JSKF,5,1,H,HMIN,HMAX,1.E-2,W)       M-605
      STOP                                                                 M-615
      END                                                                  M-620
C
      SUBROUTINE SDESOL (Y,YL,T,TEND,NY,NL,M,JSKF,MAXDER,IPRT,H,HMIN,   SDE   10
     1HMAX,RMSEPS,W)                                                    SDE   20
C                                                                       SDE   30
C     ------------------------------------------------------------------SDE   40
C                                                                       SDE   50
C     SUBROUTINE SDESOL IS A DRIVER ROUTINE FOR SUBROUTINE LDASUB.      SDE   60
C     ITS PURPOSE IS TO SET UP THE NECESSARY REFERENCES TO A LARGE      SDE   70
C     BLOCK OF AUXILLARY STORAGE, AND  OBTAIN INITIAL VALUES OF         SDE   80
C     DERIVATIVES.                                                      SDE   90
C     THE CALLING SEQUENCE FOR SDESOL IS                                SDE  100
C                                                                       SDE  110
C CALL SDESOL(Y,YL,T,TEND,NY,NL,M,JSKF,MAXDER,IPRT,H,HMIN,HMAX,RMSEPS,W)SDE  120
C                                                                       SDE  130
C     WHERE THE PARAMETERS ARE DEFINED AS FOLLOWS.                      SDE  140
C                                                                       SDE  150
C        Y         - ARRAY DIMENSIONED (7,NY).  THIS ARRAY CONTAINS THE SDE  160
C                    DEPENDENT VARIABLES AND THEIR SCALED DERIVATIVES.  SDE  170
C                    Y(J+1,I) CONTAINS THE J-TH DERIVATIVE OF THE I-TH VARSDE  180
C                    IABLE TIMES H**J/J-FACTORIAL, WHERE H IS THE CURRENT SDE  190
C                    STEPSIZE.  ON FIRST ENTRY THE CALLER SUPPLIES THE  SDE  200
C                    INITIAL VALUES OF EACH VA3IABLE IN Y(1,I).  ON SUB-  SDE  210
C                    SEQUENT ENTRIES IT IS ASSJMED THE ARRAY HAS NOT    SDE  220
C                    BEEN CHANGED.  TO INTERPOLATE TO NON-MESH POINTS,  SDE  230
C                    THESE VALJES CAN BE USED AS FOLLOWS.  IF H IS THE  SDE  240
C                    CURRENT STEPSIZE AND VALUES AT TIME  T+E  ARE      SDE  250
C                    NEEDED, LET  S = E/H  AND THEN                     SDE  260
C                                                                       SDE  270
C                                                        JS             SDE  280
C                            I-TH VARIABLE AT  T+E  IS  SUM Y(J+1,I)*S**J SDE  290
C                                                        J=0            SDE  300
C                                                                       SDE  310
C                    THE VALUE OF JS IS OBTAINED IN THE CALLING PROGRAM SDE  320
C                    BY  JS = IABS(JSKF/10)                             SDE  330
C        YL        - ARRAY OF NL VARIABLES WHICH APPEAR LINEARLY.       SDE  340
C        T         - CURRENT VALJE OF THE INDEPENDENT VARIABLE (TIME)   SDE  350
C        TEND      - END TIME                                           SDE  360
C        NY        - NJMBER OF DIFFERENTIAL EQUATIONS AND NONLINEAR     SDE  370
C                    VARIABLES.                                         SDE  380
C        NL        - NUMBER OF LINEAR VARIABLES                         SDE  390
C        M         - NJMBER OF VARIABLES INCLUJED IN THE ERROR TEST     SDE  400
C        JSKF      - AN INDICATOR USED BOTH ON INPUT AND OUTPUT         SDE  410
C                    ON INPUT, JSKF = -1 INDICATES A RESTART CALL TO    SDE  420
C                    SDESOL.  JSKF = 0 INDICATES AN INITIAL CALL TO     SDE  430
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C                 SDESOL.  JSKF > 0 INDICATES A CONTINUATION OF THE     SDE  440
C                 PREVIOUS CALL TO SDESOL.  JSKF < -1 MAY HAVE RESULTEDSDE  450
C                 FROM THE USER NEGLECTING TO TEST FOR ERROR RETURNS    SDE  460
C                 FROM SDESOL.  BECAUSE OF THIS POSSIBILITY, JSKF < -1 SDE  470
C                 RESULTS IN TERMINATION OF THE RUN WITH THE            SDE  480
C                 APPROPRIATE COMMENT.                                  SDE  490
C                 ON OUTPUT, JSKF CONSISTS OF TWO DIGITS AND SIGN,      SDE  500
C                 + OR - QP.  Q IS THE ORDER OF THE FORMULA CURRENTLY  SDE  510
C                 BEING USED.  P INDICATES THE TYPE OF RETURN, AS       SDE  520
C                 FOLLOWS.                                              SDE  530
C                 JSKF > 0, P = 1 IS THE NORMAL RETURN                  SDE  540
C                 JSKF < 0 IS AN ERROR RETURN, WITH THE FOLLOWING       SDE  550
C                 MEANINGS.                                             SDE  560
C                      P = 1  ERROR TEST FAILURE FOR H > HMIN           SDE  570
C                      P = 3  CORRECTOR FAILED TO CONVERGE FOR H > HMINSDE  580
C                      P = 4  CORRECTOR FAILED TO CONVERGE FOR FIRST    SDE  590
C                             ORDER METHOD                              SDE  600
C                      P = 5  ERROR RETURN FROM SUBROUTINE NUITSL       SDE  610
C                      P = 6  ERROR RETURN FROM SUBROUTINE DERVAL       SDE  620
C       MAXDER  - MAXIMUM ORDER DERIVATIVE THAT SHOULD BE USED IN       SDE  630
C                 METHOD.  IT MUST BE NO GREATER THAN SIX.              SDE  640
C       IPRT    - INTERNAL PRINT CONTROL INDICATOR FOR LDASUB.          SDE  650
C                      IPRT = 0  NO PRINT                               SDE  660
C                      IPRT > 0  PRINT COUNTERS, STEPSIZE, CURRENT TIMESDE  670
C                                AND VALUES OF DEPENDENT VARIABLES AT   SDE  680
C                                EACH STEP.                             SDE  690
C       H       - CURRENT STEPSIZE.  AN INITIAL VALUE MUST BE SUPPLIED SDE  700
C                 BUT NEED NOT BE THE ONE WHICH MUST BE USED, SINCE THESDE  710
C                 SUBROUTINE WILL CHOSE A SMALLER ONE IF NECCESSARY TO SDE  720
C                 KEEP THE ERROR PER STEP SMALLER THAN THE SPECIFIED    SDE  730
C                 VALUE.  IT IS BETTER TO UNDERESTIMATE THE INITIAL     SDE  740
C                 STEPSIZE THAN TO OVERESTIMATE IT.  THE STEPSIZE IS    SDE  750
C                 NORMALLY NOT CHANGED BY THE USER.                     SDE  760
C       HMIN    - MINIMUM STEPSIZE ALLOWED                              SDE  770
C       HMAX    - MAXIMUM STEPSIZE ALLOWED                              SDE  780
C       RMSEPS  - THE ERROR TEST CONSTANT.  THE ROOT-MEAN-SQUARE OF     SDE  790
C                 THE SINGLE STEP ERROR ESTIMATES,  ER(I), DIVIDED BY   SDE  800
C                 YMAX(I) = (MAXIMUM TO CURRENT TIME OF Y(I) ) MUST BE SDE  810
C                 LESS THAN EPS.  THE STEPSIZE AND/OR THE ORDER         SDE  820
C                 ARE VARIED TO ACHIEVE THIS.                           SDE  830
C       W       - SCRATCH STORAGE ARRAY.  MUST BE AT LEAST 13*NY + 5*NLSDE  840
C                 LOCATIONS, PLUS THOSE REQUIRED FOR STORAGE OF THE     SDE  850
C                 MATRIX  PW (SEE DESCRIPTION OF SUBROUTINE JACMAT).    SDE  860
C                 THE STORAGE OF PW WILL NORMALLY REQUIRE NO MORE THAN SDE  870
C                 N**2 + 2*N LOCATIONS, AND IF COMPACT STORAGE TECH-    SDE  880
C                 NIQUES ARE USED, CAN BE MUCH FEWER.                   SDE  890
C                                                                       SDE  900
C     ------------------------------------------------------------------SDE  910
```

```
      DIMENSION Y(7,1), YL(1), W(1)                                     SDE  920
      IF (JSKF.GT.0) GO TO 120                                          SDE  930
      IF (JSKF.LT.-1) GO TO 140                                         SDE  940
      N = NY+NL                                                         SDE  950
      IF (JSKF.LT.0) GO TO 110                                          SDE  960
C                                                                       SDE  970
C     IF THIS IS THE FIRST ENTRY, OBTAIN VALJES OF THE DERIVATIVES.     SDE  980
      CALL DERVAL (Y,YL,T,N,NY,W,KRETR)                                 SDE  990
      IF (KRETR.NE.0) GO TO 130                                         SDE 1000
C                                                                       SDE 1010
C     NOW SET UP STORAGE BLOCKS IN THE W ARRAY.  THIS NEEDS TO BE DONE  SDE 1020
C     ONLY INITIALLY AND ON RESTARTS.                                   SDE 1030
C                                                                       SDE 1040
C        THE ARRAY   SAVE    STARTS AT LOCATION  1     IN THE W ARRAY   SDE 1050
C        THE ARRAY   YLSV    STARTS AT LOCATION  NSVL  IN THE W ARRAY   SDE 1060
C        THE ARRAY   YMAX    STARTS AT LOCATION  NYMAX IN THE W ARRAY   SDE 1070
C        THE ARRAY   ER      STARTS AT LOCATION  NER   IN THE W ARRAY   SDE 1080
C        THE ARRAY   ESV     STARTS AT LOCATION  NESV  IN THE W ARRAY   SDE 1090
C        THE ARRAY   F1      STARTS AT LOCATION  NF1   IN THE W ARRAY   SDE 1100
C        THE ARRAY   DY      STARTS AT LOCATION  NDY   IN THE W ARRAY   SDE 1110
C        THE MATRIX  PW      STARTS AT LOCATION  NPW   IN THE W ARRAY   SDE 1120
C                                                                       SDE 1130
  110 NSVL = 7*NY+1                                                     SDE 1140
      NYMAX = NSVL+NL                                                   SDE 1150
      NER = NYMAX+NY                                                    SDE 1160
      NESV = NER+NY                                                     SDE 1170
      NF1 = NESV+NY                                                     SDE 1180
      NDY = NF1+N                                                       SDE 1190
      NPW = NDY+N                                                       SDE 1200
  120 JS = JSKF                                                         SDE 1210
      CALL LDASUB (Y,YL,T,TEND,N,NY,M,JS,KF,MAXDER,IPRT,H,HMIN,HMAX,    SDE 1220
     1RMSEPS,W,W(NSVL),W(NYMAX),W(NER),W(NESV),W(NF1),W(NDY),W(NPW))    SDE 1230
C                                                                       SDE 1240
C     CODE JSKF ON RETURN FROM LDASUB                                   SDE 1250
C                                                                       SDE 1260
      JSKF = ISIGN(JS*10+IABS(KF),KF)                                   SDE 1270
      RETURN                                                            SDE 1280
  130 JSKF = -6                                                         SDE 1290
      RETURN                                                            SDE 1300
  140 PRINT 1, JSKF                                                     SDE 1310
      STOP                                                              SDE 1320
C                                                                       SDE 1330
C                                                                       SDE 1340
    1 FORMAT ('OIT IS AN ERROR TO ENTER SDESOL WITH JSKF = ',I10//      SDE 1350
     1 '    RUN HAS BEEN TERMINATED.')                                  SDE 1360
      END                                                               SDE 1370
C
      SUBROUTINE LDASUB (Y,YL,T,TEND,N,NY,M,JSTART,KFLAG,MAXOR,IPRT,H,  LDA   10
```

```
     1HMIN,HMAX,RMSEPS,SAVE,YLSV,YMAX,ER,ESV,F1,DY,PW)                  LDA  20
C                                                                       LDA  30
C     SUBROUTINE LDASUB IS A MODIFICATION OF SUBROUTINE DFASUB          LDA  40
C     WHICH IS DUE TD R. L. BROWN AND C. W. 3EAR.  DFASUB IS DOCUMENTED LDA  50
C     IN THE REPORT                                                     LDA  60
C        DOCUMENTATI3N FOR DFASUB--                                     LDA  70
C        BY R. L. BROWN AND C. W. GEAR                                  LDA  80
C        REPORT UIUCDCS-R-73-575, JULY 1973                             LDA  90
C        UNIVERSITY 3F ILLINOIS AT URBANA-CHAMPAIGN                     LDA 100
C        URBANA, ILLINOIS  61801                                        LDA 110
C     THIS REP3RT IS AVAILABLE FROM THE NATI3NAL TECHNICAL INFORMATION  LDA 120
C     SERVICE OF THE U. S. DEPARTMENT OF COMMERCE UNDER ACCESSION NUMBERLDA 130
C     COO-1469-225.                                                     LDA 140
C                                                                       LDA 150
C     THE MODIFICATI3N HERE IS D3CUMENTED IN THE REPORT                 LDA 160
C        A PROGRAM F3R THE NUMERICAL SOLUTIOV OF LARGE SPARSE SYSTEMS OFLDA 170
C        ALGEBRAIC AVD IMPLICITLY DEFINED STIFF DIFFERENTIAL EQUATIONS  LDA 180
C        BY RICHARD FRANKE                                              LDA 190
C        REPORT NPS53FE76051, MAY 1976                                  LDA 200
C        NAVAL POSTGRADUATE SCHOOL                                      LDA 210
C        MONTEREY, CALIFORNIA  93940                                    LDA 220
C                                                                       LDA 230
C     ------------------------------------------------------------------LDA 240
C                                                                       LDA 250
C     THE CALLING SEQJENCE FOR LDASUB IS                                LDA 260
C                                                                       LDA 270
C     CALL LDASUB(Y,YL,T,TEND,V,NY,M,JSTART,KFLAG,MAXOR,IPRT,H,HMIN,    LDA 280
C     HMAX,RMSEPS,SAVE,YLSV,YMAX,ER,ESV,F1,DY,PW)                       LDA 290
C                                                                       LDA 300
C     WHERE THE PARAMETERS ARE DEFINED AS FOLLOWS.                      LDA 310
C        Y        - ARRAY DIMENSI3NED (7,NY).  THIS ARRAY CONTAINS THE  LDA 320
C                   DEPENDENT VARIABLES AND THEIR SCALED DERIVATIVES.   LDA 330
C                   Y(J+1,I) CONTAINS THE J-TH DERIVATIVE OF THE I-TH VARLDA 340
C                   IABLE TIMES H**J/J-FACTORIAL, WHERE H IS THE CURRENT LDA 350
C                   STEPSIZE.  ON FIRST ENTRY THE CALLER SUPPLIES THE   LDA 360
C                   IVITIAL VALUES 3F EACH VA3IABLE IN Y(1,I) AND AN    LDA 370
C                   ESTIMATE OF THE INITIAL VALUES OF THE DERIVATIVES   LDA 380
C                   IV Y(2,I).  ON SUBSEQUENT ENTRIES IT IS ASSUMED THAT LDA 390
C                   THE ARRAY HAS NOT BEEN CHANGED.  TO INTERPOLATE TO  LDA 400
C                   NOV-MESH POINTS, THESE VALUES CAN BE USED AS FOLLOWS.LDA 410
C                   IF H IS THE CURRENT STEPSIZE AND VALUES AT TIME T+E LDA 420
C                   NEEDED, LET S = E/H  AND THEN                       LDA 430
C                                                                       LDA 440
C                                                 NQ                    LDA 450
C                           I-TH VARIABLE AT  T+E IS  SUM Y(J+1,I)*S**J LDA 460
C                                                 J=0                   LDA 470
C                                                                       LDA 480
C                   THE VALJE OF NQ IS OBTAIVED IN THE CALLING PROGRAM  LDA 490
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C                 BY  NQ = JSTART.                                     LDA  500
C                                                                      LDA  510
C        YL     - ARRAY OF NL = N - NY VARIABLES WHICH APPEAR LINEARLY.LDA  520
C                 THE USER SUPPLIES INITIAL VALUES FOR THESE VARIABLES.LDA  530
C        T      - CURRENT VALUE OF THE INDEPENDENT VARIABLE (TIME)     LDA  540
C        TEND   - END TIME                                             LDA  550
C        N      - TOTAL NUMBER OF VARIABLES                            LDA  560
C        NY     - NUMBER OF DIFFERENTIAL EQUATIONS AND NONLINEAR       LDA  570
C                 VARIABLES.                                           LDA  580
C        M      - NUMBER OF VARIABLES INCLUDED IN THE ERROR TEST.      LDA  590
C                 THIS NUMBER CAN BE NO GREATER THAN NY.  IF IT IS     LDA  600
C                 GREATER THAN NY, NY VARIABLES ARE USED IN THE ERROR  LDA  610
C                 TEST.                                                LDA  620
C        JSTART - INPUT AND OUTPUT INDICATOR.                          LDA  630
C                 ON INPUT JSTART HAS THE FOLLOWING MEANINGS.          LDA  640
C                      <0  THIS INDICATES A RE-START FROM A PREVIOUS   LDA  650
C                          POINT FOLLOWING TERMINATION OF THE RUN OR   LDA  660
C                          SOLUTION OF ANOTHER PROBLEM DURING THE SAME LDA  670
C                          RUN.  PARAMETERS IN THE CALLING SEQUENCE    LDA  680
C                          MUST HAVE BEEN PRESERVED FROM THE PREVIOUS  LDA  690
C                          USE, PARTICULARLY THE ARRAYS                LDA  700
C                               SAVE, YLSV, ESV, AND PW.               LDA  710
C                          THESE ARRAYS MUST BE SAVED AFTER A CALL     LDA  720
C                          TO SUBROUTINE LDASAV, WHICH ALSO SAVES      LDA  730
C                          NECESSARY PARAMETERS INTERNAL TO LDASUB.    LDA  740
C                      =0  INDICATES AN INITIAL CALL TO LDASUB.  THE   LDA  750
C                          ROUTINE INITIALIZES ITSELF, SCALES THE      LDA  760
C                          DERIVATIVES IN Y(2,I) AND THEN PERFORMS THE LDA  770
C                          INTEGRATION UNTIL T > TEND.                 LDA  780
C                      >0  INDICATES THE SOLUTION IS TO BE CONTINUED.  LDA  790
C                          AFTER THE INITIAL ENTRY IT IS NEITHER       LDA  800
C                          DESIRABLE NOR NECESSARY TO RE-ENTER WITH    LDA  810
C                          JSTART = 0, SINCE THIS RE-INITIALIZES       LDA  820
C                          THE CODE, BEGINNING WITH A FIRST ORDER      LDA  830
C                          METHOD AGAIN.                               LDA  840
C                 ON OUTPUT, JSTART IS SET TO THE VALUE OF NQ, THE     LDA  850
C                 ORDER OF THE FORMULA CURRENTLY BEING USED.           LDA  860
C        KFLAG  - THE COMPLETION CODE INDICATOR, WITH THE FOLLOWING    LDA  870
C                 MEANINGS                                             LDA  880
C                      +1  THE INTEGRATION WAS SUCCESSFUL              LDA  890
C                      -1  ERROR TEST FAILURE FOR H > HMIN             LDA  900
C                      -3  CORRECTOR FAILED TO CONVERGE FOR H > HMIN   LDA  910
C                      -4  CORRECTOR FAILED TO CONVERGE FOR FIRST      LDA  920
C                          ORDER METHOD                                LDA  930
C                      -5  ERROR RETURN FROM SUBROUTINE NUITSL         LDA  940
C        MAXOR  - MAXIMUM ORDER DERIVATIVE THAT SHOULD BE USED IN THE  LDA  950
C                 METHOD.  IT MUST BE NO GREATER THAN SIX.  IF IT IS   LDA  960
C                 GREATER THAN SIX, THE MAXIMUM ORDER USED WILL BE SIX.LDA  970
```

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C        IPRT     - INTERNAL PRINT CONTROL INDICATOR                   LDA  980
C                       = 0  NO PRINT                                  LDA  990
C                       > 0  PRINT COUNTERS, STEPSIZE, CURRENT TIME    LDA 1000
C                            AND VALUES OF DEPENDENT VARIABLES AT      LDA 1010
C                            EACH STEP.                                LDA 1020
C        H        - CURRENT STEPSIZE.  AN INITIAL VALUE MUST BE SUPPLIED LDA 1030
C                   BUT NEED NOT BE THE ONE WHICH WILL BE USED, SINCE THELDA 1040
C                   SUBROUTINE WILL CHOOSE A SMALLER ONE IF NECCESSARY TOLDA 1050
C                   KEEP THE ERROR PER STEP SMALLER THAN THE SPECIFIED  LDA 1060
C                   VALUE.  IT IS BETTER TO UNDERESTIMATE THE INITIAL   LDA 1070
C                   STEPSIZE THAN TO OVERESTIMATE IT.  THE STEPSIZE IS  LDA 1080
C                   NORMALLY NOT CHANGED BY THE USER.                  LDA 1090
C        HMIN     - MINIMUM STEPSIZE ALLOWED                           LDA 1100
C        HMAX     - MAXIMUM STEPSIZE ALLOWED                           LDA 1110
C        RMSEPS   - THE ERROR TEST CONSTANT.  THE ROOT-MEAN-SQUARE OF  LDA 1120
C                   THE SINGLE STEP ERROR ESTIMATES,  ER(I), DIVIDED BY LDA 1130
C                   YMAX(I) = (MAXIMUM TO CURRENT TIME OF Y(I)) MUST BE LDA 1140
C                   LESS THAN RMSEPS.  THE STEPSIZE AND/OR ORDER ARE   LDA 1150
C                   VARIED TO ACHIEVE THIS.                            LDA 1160
C        SAVE     - AN ARRAY OF LENGTH AT LEAST 7*NY                   LDA 1170
C        YLSV     - AN ARRAY OF LENGTH AT LEAST NL                     LDA 1180
C        YMAX     - A VECTOR OF LENGTH NY WHICH CONTAINS THE MAXIMUM   LDA 1190
C                   OF EACH Y SEEN SO FAR.  ON THE FIRST CALL, THESE WILLLDA 1200
C                   BE INITIALIZED AS YMAX(I) = MAX(1,|Y(1,I)|)        LDA 1210
C        ER       - A VECTOR OF LENGTH NY                              LDA 1220
C        ESV      - A VECTOR OF LENGTH NY                              LDA 1230
C        F1       - A VECTOR OF LENGTH N = NY + NL                     LDA 1240
C        DY       - A VECTOR OF LENGTH N = NY + NL                     LDA 1250
C        PW       - AN ARRAY IN WHICH THE  J  MATRIX COMPUTED          LDA 1260
C                   IN SUBROUTINE JACMAT WILL BE STORED.  SIZE WHICH   LDA 1270
C                   MUST BE ALLOWED IS DETERMINED BY THE STORAGE TECH- LDA 1280
C                   NIQUE USED FOR IT, BUT NORMALLY WON'T BE MORE THAN LDA 1290
C                   N**2 + 2*N LOCATIONS, THE LATTER 2*N BEING REQUIRED LDA 1300
C                   BY THE LINEAR EQUATION SOLVER.                     LDA 1310
C                                                                      LDA 1320
C     -----------------------------------------------------------------LDA 1330
      DIMENSION Y(7,1), YL(1), SAVE(7,1), YMAX(1), ER(1), YLSV(1), F1(1)LDA 1340
     1, PERT(6,3), COF(21), ESV(1), DY(1), PW(1), SAV(1), A(29)        LDA 1350
      EQUIVALENCE (A(8),BND), (A(9),BR), (A(10),E), (A(11),EDWN),      LDA 1360
     1(A(12),ENQ1), (A(13),ENQ2), (A(14),ENQ3), (A(15),EPS), (A(16),EUP)LDA 1370
     2,(A(17),HNEW), (A(18),PEPSH), (A(19),IDOUB), (A(20),IWEVAL),     LDA 1380
     3(A(21),K), (A(22),LCOPYL), (A(23),LCOPYY), (A(24),MAXDER),       LDA 1390
     4(A(25),M1), (A(26),NL), (A(27),NQ), (A(28),NS), (A(29),NW)       LDA 1400
C                                                                      LDA 1410
C     -----------------------------------------------------------------LDA 1420
C                                                                      LDA 1430
C     THE COEFFICIENTS IN THE PERT ARRAY ARE USED FOR ERROR TESTING AND LDA 1440
C     CHANGING STEPSIZE AND NEED TO BE ACCURATE TO ONLY A FEW DIGITS.  LDA 1450
```

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C
C     ------------------------------------------------------------------
      DATA PERT/4.,9.,16.,25.,36.,49.,9.,16.,25.,36.,49.,64.,1.,1.,.25,
     12.7889E-2,1.70569E-3,6.83929E-5/
C     ------------------------------------------------------------------
C
C     THE ENTRIES IN THE COF ARRAY ARE THE CJEFFICIENTS FOR THE STIFFLY
C     STABLE METHODS JSED IN THIS PROGRAM AND ARE TO BE THE MACHINE
C     PRECISION EQUIVALENTS OF THE FOLLOWING CONSTANTS.
C
C     -1
C     -3/2 , -1/2
C     -11/6 , -1 , -1/6
C     -25/12 , -35/24 , -5/12 , -1/24
C     -137/60 , -15/8 , -17/24 , -1/8 , -1/120
C     -147/60, -203/90 , -49/48 , -35/144 , -7/240 , -1/720
C
C     ------------------------------------------------------------------
      DATA CJF/-1.,-1.5,-.5,-1.833333,-1.,-.1666667,-2.083333,-1.458333,
     1-.4166667,-.04166667,-2.283333,-1.875,-.7083333,-.125,-.008333333,
     2-2.45,-2.255556,-1.020833,-.2430556,-.J2916667,-.001388889/
      IF (JSTART) 1J0,110,150
C     ------------------------------------------------------------------
C     IF THIS IS A RESTART ENTRY, RESTORE Y AND YL FROM THE SAVE AND
C     YLSV ARRAYS, WHERE THEY WERE SAVED BY A PREVIOUS CALL TO LDASAV.
C     ------------------------------------------------------------------
  100 CALL COPYZ (Y,SAVE,L:OPYY)
      CALL COPYZ (YL,YLSV,LCOPYL)
      GO TO 150
C     ------------------------------------------------------------------
C     IF THIS IS THE FIRST CALL, INITIALIZE YMAX, SCALE DERIVATIVES, AND
C     INITIALIZE INDI:ATORS AND SET ORDER TO ONE.
C     FOR DOUBLE PRECISION, SET LCOPYL = 14*\Y AND LCOPYL = 2*NL IF
C     SUBROUTINE COPYZ IS IN SINGLE PRECISIO\.
C     ------------------------------------------------------------------
  110 NL = N-NY
      LCOPYY = 7*NY
      LCOPYL = NL
      M1 = MINO(M,NY)
      EPS = SQRT(FLJAT(M1))*RMSEPS
      MAXDER = MINO(MAXOR,6)
      IF (IPRT.LE.0) GJ TO 120
      PRINT 3, N,NL,RMSEPS,TEND,H
      PRINT 4
  120 NS = 0
      NW = 0
C
      DO 130 J=1,NY
```

```
      YMAX(J) = AMAX1(1.,ABS(Y(1,J)))                                   LDA 1940
  130 Y(2,J) = Y(2,J)*H                                                 LDA 1950
C                                                                       LDA 1960
      NQ = 1                                                            LDA 1970
      BR = 1.                                                           LDA 1980
      ASSIGN 190 TO IRET                                                LDA 1990
C ----------------------------------------------------------------------LDA 2000
C     SET COEFFICIENTS FOR THE ORDER CURRENTLY BEING USED.              LDA 2010
C     E IS A TEST FOR ERRORS OF THE CURRENT ORDER NQ                    LDA 2020
C     EUP IS TO TEST FOR INCREASING THE ORDER, EDWN FOR DECREASING THE  LDA 2030
C     ORDER.                                                            LDA 2040
C ----------------------------------------------------------------------LDA 2050
  140 K = NQ*(NQ-1)/2                                                   LDA 2060
      CALL COPYZ (A(2),COF(K+1),NQ)                                     LDA 2070
      K = NQ+1                                                          LDA 2080
      IDOUB = NQ                                                        LDA 2090
      ENQ1 = .5/NQ                                                      LDA 2100
      ENQ2 = .5/K                                                       LDA 2110
      ENQ3 = .5/(NQ+2)                                                  LDA 2120
      PEPSH = EPS**2                                                    LDA 2130
      E = PERT(NQ,1)*PEPSH                                              LDA 2140
      EUP = PERT(NQ,2)*PEPSH                                            LDA 2150
      EDWN = PERT(NQ,3)*PEPSH                                           LDA 2160
      BND = (EPS*ENQ3)**2                                               LDA 2170
      IWEVAL = 1                                                        LDA 2180
      GO TO IRET, (190,200,490,570)                                     LDA 2190
  150 IF (H.EQ.HNEW) GO TO 190                                          LDA 2200
C ----------------------------------------------------------------------LDA 2210
C     IF CALLER HAS CHANGED H, RESCALE DERIVATIVES TO REFLECT THAT HNEW LDA 2220
C     WAS USED ON THE LAST CALL.                                        LDA 2230
C ----------------------------------------------------------------------LDA 2240
      R = H/HNEW                                                        LDA 2250
      ASSIGN 190 TO IRET                                                LDA 2260
      GO TO 610                                                         LDA 2270
C ----------------------------------------------------------------------LDA 2280
C     SET JSTART TO NQ, THE CURRENT ORDER OF THE METHOD, BEFORE EXIT,   LDA 2290
C     AND SAVE THE CURRENT STEPSIZE IN HNEW.                            LDA 2300
C ---------------------------------------------------------------------- LDA 2310
  160 JSTART = NQ                                                       LDA 2320
      HNEW = H                                                          LDA 2330
      RETURN                                                            LDA 2340
  170 NS = NS+1                                                         LDA 2350
      IF (IPRT.LE.0) GO TO 180                                          LDA 2360
C ----------------------------------------------------------------------LDA 2370
C     PRINT DATA IF DESIRED BY USER                                     LDA 2380
C ----------------------------------------------------------------------LDA 2390
      PRINT 1, NS,NW,NQ,H,T,(Y(1,I),I=1,NY)                             LDA 2400
      IF (NL.GT.0) PRINT 2, (YL(I),I=1,NL)                              LDA 2410
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```
  180 CONTINUE                                                          LDA 2420
      IF (KFLAG.LT.0) GO TO 160                                         LDA 2430
      IF (T.GE.TEND) GO TO 160                                          LDA 2440
C     ------------------------------------------------------------------LDA 2450
C     TAKE ANOTHER STEP IF T < TEND                                     LDA 2460
C     ------------------------------------------------------------------LDA 2470
      JSTART = 1                                                        LDA 2480
C     ------------------------------------------------------------------LDA 2490
C     SAVE DATA FOR TRIAL WITH A SMALLER TIMESTEP IF THIS STEP FAILS    LDA 2500
C     ------------------------------------------------------------------LDA 2510
  190 CALL COPYZ (SAVE,Y,LCOPYY)                                        LDA 2520
      CALL COPYZ (YLSV,YL,LCOPYL)                                       LDA 2530
      RACUM = 1.                                                        LDA 2540
      KFLAG = 1                                                         LDA 2550
      HOLD = H                                                          LDA 2560
      NQOLD = NQ                                                        LDA 2570
      TOLD = T                                                          LDA 2580
  200 T = T+H                                                           LDA 2590
      HINV = 1./H                                                       LDA 2600
C     ------------------------------------------------------------------LDA 2610
C     COMPUTE PREDICTED VALUES BY EFFECTIVELY MULTIPLYING DERIVATIVE    LDA 2620
C     VECTOR BY PASCAL TRIANGLE MATRIX                                  LDA 2630
C     ------------------------------------------------------------------LDA 2640
C                                                                       LDA 2650
      DO 210 J=2,K                                                      LDA 2660
      J3 = K+J-1                                                        LDA 2670
C                                                                       LDA 2680
      DO 210 J1=J,K                                                     LDA 2690
      J2 = J3-J1                                                        LDA 2700
C                                                                       LDA 2710
      DO 210 I=1,NY                                                     LDA 2720
  210 Y(J2,I) = Y(J2,I)+Y(J2+1,I)                                       LDA 2730
C                                                                       LDA 2740
C                                                                       LDA 2750
      DO 220 I=1,NY                                                     LDA 2760
  220 ER(I) = 0.                                                        LDA 2770
C                                                                       LDA 2780
C     ------------------------------------------------------------------LDA 2790
C     DO UP TO THREE CORRECTOR ITERATIONS.  CONVERGENCE IS OBTAINED WHENLDA 2800
C     CHANGES ARE LESS THAN BND WHICH IS DEPENDENT ON THE ERROR TEST    LDA 2810
C     CONSTANT.  THE SUM OF CORRECTIONS IS ACCUMULATED IN ER(I).  IT IS LDA 2820
C     EQUAL TO THE K-TH DERIVATIVE OF Y TIMES H**K/(K-FACTORIAL*A(K)),  LDA 2830
C     AND THUS IS PROPORTIONAL TO THE ACTUAL ERRORS TO THE LOWEST POWER LDA 2840
C     OF H PRESENT, WHICH IS H**K.                                      LDA 2850
C     ------------------------------------------------------------------LDA 2860
C                                                                       LDA 2870
      DO 270 L=1,3                                                      LDA 2880
      CALL DIFFUN (Y,YL,T,HINV,DY)                                      LDA 2890
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      IF (IWEVAL.LT.1) GO TO 230                                        LDA 2900
C     ------------------------------------------------------------------LDA 2910
C     IF THERE HAS BEEN A CHANGE OF ORDER OR THERE HAS BEEN TROUBLE     LDA 2920
C     WITH CONVERGENCE, PW IS RE-EVALUATED PRIOR TO STARTING THE        LDA 2930
C     CORRECTOR ITERATION.  IWEVAL IS THEN SET TO -1 AS AN INDICATOR    LDA 2940
C     THAT IT HAS BEEN DONE.  NEWPW IS SET NONZERO TO INDICATE TO       LDA 2950
C     SUBROUTINE NUITSL THAT A NEW PW HAS BEEN PROVIDED.                LDA 2960
C     ------------------------------------------------------------------LDA 2970
C     ------------------------------------------------------------------LDA 3250
      CALL JACMAT (Y,YL,T,HINV,A(2),N,NY,EPS,DY,F1,PW)                  LDA 2980
      KFLAG = 1                                                         LDA 2990
      IWEVAL = -1                                                       LDA 3000
      NW = NW+1                                                         LDA 3010
      NEWPW = 1                                                         LDA 3020
  230 CALL NUITSL (PW,DY,F1,N,NY,EPS,YMAX,NEWPW,KRRET)                  LDA 3030
      IF (KRRET.NE.0) GO TO 600                                         LDA 3040
      IF (NL.LE.0) G3 TO 250                                            LDA 3050
C                                                                       LDA 3060
      DO 240 I=1,NL                                                     LDA 3070
  240 YL(I) = YL(I)-F1(I+NY)                                            LDA 3080
C                                                                       LDA 3090
  250 CONTINUE                                                          LDA 3100
      DEL = 0.                                                          LDA 3110
C                                                                       LDA 3120
      DO 260 I=1,NY                                                     LDA 3130
      Y(1,I) = Y(1,I)-F1(I)                                             LDA 3140
      Y(2,I) = Y(2,I)+A(2)*F1(I)                                        LDA 3150
      ER(I) = ER(I)+F1(I)                                               LDA 3160
      DEL = DEL+(F1(I)/AMAX1(YMAX(I),ABS(Y(1,I))))**2                   LDA 3170
  260 CONTINUE                                                          LDA 3180
C                                                                       LDA 3190
      IF (L.GE.2) BR = AMAX1(.9*BR,DEL/DEL1)                            LDA 3200
      DEL1 = DEL                                                        LDA 3210
      IF (AMIN1(DEL,BR*DEL*2.).LE.BND) GO TO 330                        LDA 3220
  270 CONTINUE                                                          LDA 3230
C                                                                       LDA 3240
C     THE CORRECTIJR ITERATION FAILED TO CONVERGE IN 3 TRIES.  VARIOUS  LDA 3260
C     POSSIBILITIES ARE CHECKED FOR.  IF H IS ALREADY HMIN AND PW HAS   LDA 3270
C     ALREADY BEEN RE-EVALUATED, A NO CONVERGENCE EXIT IS TAKEN.        LDA 3280
C     OTHERWISE THE MATRIX PW IS RE-EVALUATED AND/OR (IN THAT ORDER) THELDA 3290
C     STEP IS REDJCED TO TRY AND GET CONVERGENCE.                       LDA 3300
C     ------------------------------------------------------------------LDA 3310
      T = TOLD                                                          LDA 3320
      IF (IWEVAL) 280,300,290                                           LDA 3330
  280 IF (H.LE.HMIN*1.00001) GO TO 310                                  LDA 3340
  290 RACUM = RACUM*.25                                                 LDA 3350
  300 CONTINUE                                                          LDA 3360
      GO TO 560                                                         LDA 3370
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  310 KFLAG = -3                                                        LDA 3380
C     ------------------------------------------------------------------LDA 3390
C     RESTORE Y AND YL AFTER CONVERGENCE FAILURE                        LDA 3400
C     ------------------------------------------------------------------LDA 3410
  320 CALL COPYZ (Y,SAVE,LCOPYY)                                        LDA 3420
      CALL COPYZ (YL,YLSV,LCOPYL)                                       LDA 3430
      H = HOLD                                                          LDA 3440
      NQ = NQOLD                                                        LDA 3450
      GO TO 170                                                         LDA 3460
C     ------------------------------------------------------------------LDA 3470
C     THE C3RRECTOR :ONVERGED, SO NOW THE ER3OR TEST IS MADE.           LDA 3480
C     ------------------------------------------------------------------LDA 3490
  330 D = 0.                                                            LDA 3500
C                                                                       LDA 3510
      DO 340 I=1,M1                                                     LDA 3520
      YM = AMAX1(ABS(Y(1,I)),YMAX(I))                                   LDA 3530
  340 D = D+(ER(I)/YM)**2                                               LDA 3540
C                                                                       LDA 3550
      IWEVAL = 0                                                        LDA 3560
      IF (D.GT.E) GO TO 380                                             LDA 3570
C     ------------------------------------------------------------------LDA 3580
C     THE ERROR TEST IS OKAY, SO THE STEP IS ACCEPTED.  IF IDOUB        LDA 3590
C     NOW BECOMES NEGATIVE, A TEST IS MADE TJ SEE IF THE STEP SIZE      LDA 3600
C     CAN BE INCREAS=D AT THIS ORDER OR ONE 4IGHER OR ONE LOWER.        LDA 3610
C     THE CHANGE IS MADE ONLY IF THE STEP CAV BE INCREASED BY AT        LDA 3620
C     LEAST 10%.  ID3UB IS SET TO NQ TO PREVENT FURTHER TESTING         LDA 3630
C     FOR A WHILE.  IF NO CHANGE IS MADE, ID3UB IS SET TO 9.            LDA 3640
C     ------------------------------------------------------------------LDA 3650
      IF (K.LT.3) G3 TO 360                                             LDA 3660
C                                                                       LDA 3670
      DO 350 J=3,K                                                      LDA 3680
C                                                                       LDA 3690
      DO 350 I=1,NY                                                     LDA 3700
  350 Y(J,I) = Y(J,I)+A(J)*ER(I)                                        LDA 3710
C                                                                       LDA 3720
  360 KFLAG = 1                                                         LDA 3730
      IDOUB = IDOUB-1                                                   LDA 3740
      IF (IDOUB) 410,370,510                                            LDA 3750
  370 CALL COPYZ (ESV,ER,M1)                                            LDA 3760
      GO T3 510                                                         LDA 3770
C     ------------------------------------------------------------------LDA 3780
C     THE ERROR TEST FAILED.  IF JSTART = 0, THE DERIVATIVES IN THE     LDA 3790
C     SAVE A3RAY ARE UPDATED.  TESTS ARE THEV MADE TO FIX THE STEPSIZE  LDA 3800
C     AND PERHAPS REDJCE THE ORDER.  AFTER RESTORING AND SCALING THE    LDA 3810
C     Y VARIABLES, THE STEP IS RETRIED.                                 LDA 3820
C     ------------------------------------------------------------------LDA 3830
C     ------------------------------------------------------------------LDA 4300
  380 IF (JSTART.GT.0) GO TO 400                                        LDA 3840
```

```
C                                                                        LDA 3850
      DO 390 I=1,NY                                                      LDA 3860
  390 SAVE(2,I) = Y(2,I)                                                 LDA 3870
C                                                                        LDA 3880
  400 KFLAG = KFLAG-2                                                    LDA 3890
      IF (H.LE.HMIN) GO TO 550                                           LDA 3900
      T = TOLD                                                           LDA 3910
      IF (KFLAG.LE.-5) GO TO 530                                         LDA 3920
  410 PR2 = (D/E)**ENQ2*1.2                                              LDA 3930
      L = 0                                                              LDA 3940
      IF (NQ.LE.1) GO TO 430                                             LDA 3950
      D = 0.                                                             LDA 3960
C                                                                        LDA 3970
      DO 420 J=1,M1                                                      LDA 3980
      YM = AMAX1(ABS(Y(1,J)),YMAX(J))                                    LDA 3990
  420 D = D+(Y(K,J)/YM)**2                                               LDA 4000
C                                                                        LDA 4010
      PR1 = (D/EDWN)**ENQ1*1.3                                           LDA 4020
      IF (PR1.GE.PR2) GO TO 430                                          LDA 4030
      PR2 = PR1                                                          LDA 4040
      L = -1                                                             LDA 4050
  430 IF (KFLAG.LT.0.OR.NQ.GE.MAXDER) GO TO 450                          LDA 4060
      D = 0                                                              LDA 4070
C                                                                        LDA 4080
      DO 440 J=1,M1                                                      LDA 4090
      YM = AMAX1(ABS(Y(1,J)),YMAX(J))                                    LDA 4100
  440 D = D+((ER(J)-ESV(J))/YM)**2                                       LDA 4110
C                                                                        LDA 4120
      PR1 = (D/EUP)**ENQ3*1.4                                            LDA 4130
      IF (PR1.GE.PR2) GO TO 450                                          LDA 4140
      PR2 = PR1                                                          LDA 4150
      L = 1                                                              LDA 4160
  450 R = 1./AMAX1(PR2,1.E-5)                                            LDA 4170
      IF (KFLAG.LT.0.OR.R.GE.1.1) GO TO 460                              LDA 4180
      IDOUB = 9                                                          LDA 4190
      GO TO 510                                                          LDA 4200
  460 NEWQ = NQ+L                                                        LDA 4210
      K = NEWQ+1                                                         LDA 4220
      IF (NEWQ.LE.NQ) GO TO 480                                          LDA 4230
      R1 = A(NEWQ)/FLOAT(NEWQ)                                           LDA 4240
C                                                                        LDA 4250
      DO 470 J=1,NY                                                      LDA 4260
  470 Y(K,J) = ER(J)*R1                                                  LDA 4270
C                                                                        LDA 4280
  480 CONTINUE                                                           LDA 4290
C     IF THE STEP WAS OKAY, SCALE THE Y VARIABLES IN ACCORDANCE          LDA 4310
C     WITH THE NEW VALUE OF H.  IF KFLAG < 0, HOWEVER, USE THE           LDA 4320
C     SAVED VALUES (IN SAVE AND YLSV).  IN EITHER CASE, IF THE ORDER     LDA 4330
```

```
C      HAS CHANGED IT IS NECESSARY TO FIX CERTAIN PARAMETERS BY CALLING
C      THE PROGRAM SEGMENT AT STATEMENT NUMBER 140.
C      ----------------------------------------------------------------
       IDOUB = NQ
       IF (NEWQ.EQ.NQ) GO TO 490
       NQ = NEWQ
       ASSIGN 490 TO IRET
       GO TO 140
  490  IF (KFLAG.GT.0) GO TO 500
       RACUM = RACUM*R
       GO TO 560
  500  R = AMAX1(AMIN1(HMAX/H,R),HMIN/H)
       H = H*R
       IWEVAL = 1
       ASSIGN 510 TO IRET
       GO TO 610
C
  510  DO 520 I=1,M1
  520  YMAX(I) = AMAX1(ABS(Y(1,I)),YMAX(I))
C
       GO TO 170
C      ----------------------------------------------------------------
C      THE ERROR TEST HAS NOW FAILED THREE TIMES, SO THE DERIVATIVES ARE
C      IN BAD SHAPE.  RETURN TO FIRST ORDER METHOD AND TRY AGAIN.  OF
C      COURSE, IF NQ = 1 ALREADY, THEN THERE IS NO HOPE AND WE EXIT WITH
C      KFLAG = -4.
C      ----------------------------------------------------------------
  530  IF (NQ.EQ.1) GO TO 540
       NQ = 1
       IDOUB = 1
       ASSIGN 570 TO IRET
       GO TO 140
  540  NQOLD = 1
       KFLAG = -4
       GO TO 320
  550  KFLAG = -1
       GO TO 170
C      ----------------------------------------------------------------
C      THIS SECTION RESTORES THE SAVED VALUES OF Y AND YL, SCALING THE
C      Y DERIVATIVES AS NECESSARY, AND THEN RETURNS TO THE PREDICTOR LOOP
C      ----------------------------------------------------------------
  560  H = HOLD*RACUM
       H = AMAX1(HMIN,AMIN1(H,HMAX))
  570  RACUM = H/HOLD
       R1 = 1.
C
       DO 580 J=2,K
       R1 = R1*RACUM
```

```
C
      DO 580 I=1,NY
  580 Y(J,I) = SAVE(J,I)*R1
C
C
      DO 590 I=1,NY
  590 Y(1,I) = SAVE(1,I)
C
      CALL COPYZ (YL,YLSV,LCOPYL)
      IWEVAL = 1
      GO TO 200
  600 KFLAG = -5
      GO TO 160
C     -------------------------------------------------------------------
C     THIS SECTION SCALES THE Y DERIVATIVES BY R**J
C     ------------------------------------------------------------------
  610 R1 = 1.
C
      DO 620 J=2,K
      R1 = R1*R
C
      DO 620 I=1,NY
  620 Y(J,I) = Y(J,I)*R1
C
      GO TJ IRET, (190,510)
C     -------------------------------------------------------------------
C     THIS SECTION ALLOWS FOR RESTARTS AFTER SOLVING ANOTHER PROBLEM, OR
C     HAVING TERMINATED THE CURRENT COMPUTER RUN.  SUBROUTINE LDASAV
C     SAVES THE NECESSARY VALUES WHICH ARE IVTERNAL TO LDASUB.  FOR
C     DOUBLE PRECISIJN, WITH COPYZ IN SINGLE PRECISION, THE NUMBER OF
C     LOCATIONS TO 3E SAVED AND RESTORED, LCJPYS AND LCOPYR, MUST BE
C     SET TO 58.
C     IT IS ASSUMED THAT IN ADDITION TO THE VARIABLES IN THE ARRAY A
C     SAVED BY CALLING LDASAV, THE USER ALSO SAVES THE ARRAYS SAVE,
C     YLSV, YMAX, ESV, AND PW.
C
C     TO RESTART THE USER FIRST CALLS LDARST TO RESTORE THE VALUES SAVED
C     BY LDASAV, THEV RE-ENTERS LDASUB WITH JSTART < 0, AND WITH THE
C     OTHER PARAMETERS THE SAME AS RETURNED FROM THE LAST ENTRY TO
C     LDASUB, PARTICJLARLY THOSE ARRAYS MENTIONED ABOVE.
C     -------------------------------------------------------------------
      ENTRY LDASAV(SAV)
      LCOPYS = 29
      CALL COPYZ (SAV,A,LCOPYS)
      CALL COPYZ (SAVE,Y,LCOPYY)
      CALL COPYZ (YLSV,YL,LCOPYL)
      RETURN
C
```

```
      ENTRY LDARST(SAV)                                                 LDA 5300
      LCOPYR = 29                                                       LDA 5310
      CALL COPYZ (A,SAV,LCOPYR)                                         LDA 5320
      RETURN                                                            LDA 5330
C                                                                       LDA 5340
C     ------------------------------------------------------------------LDA 5350
C                                                                       LDA 5360
C                                                                       LDA 5370
    1 FORMAT (2I5,I2,1P2E10.2,7E14.6/(32X,7E14.6))                      LDA 5380
    2 FORMAT (32X,1P7E14.6)                                             LDA 5390
    3 FORMAT ('1  N =',I3,'    NL =',I3,'    RMSEPS =',1PE9.2,'    TEND =' LDA 5400
     1   ,E9.2,'   H =',E9.2//)                                         LDA 5410
    4 FORMAT ('    NS    NW Q     H',8X,'T     ',8X,'Y(1,*) AND YL(*)'//)   LDA 5420
      END                                                               LDA 5430
C
      SUBROUTINE COPYZ(S,Y,L)                                           COP   10
      DIMENSION S(1),Y(1)                                               COP   20
C     ------------------------------------------------------------------COP   30
C                                                                       COP   40
C     THIS SUBROUTINE COPIES THE ARRAY Y, OF LENGTH L, INTO THE ARRAY S COP   50
C                                                                       COP   60
C     ------------------------------------------------------------------COP   70
      IF(L.LE.0)RETJRN                                                  COP   80
      DO 100 J=1,L                                                      COP   90
  100 S(J) = Y(J)                                                       COP  100
      RETURN                                                            COP  110
      END                                                               COP  120
C
      SUBROUTINE DERVAL (Y,YL,T,N,NY,W,KERET)                           DER   10
C                                                                       DER   20
C     THIS SUBROUTINE CALCULATES THE INTIAL VALUES OF THE DERIVATIVES   DER   30
C     IN THE GENERAL CASE.  IT IS WRITTEN SO THAT IT SHOULD WORK IF THE DER   40
C     FIRST NY EQUATIONS ALL INVOLVE DERIVATIVES.  IT ATTEMPTS TO SOLVE DER   50
C     THE FIRST NY EQUATIONS USING NEWTON'S METHOD, BUT SINCE IT TRIES  DER   60
C     TO EVALUATE DF/DY' BY CALLING JACMAT IN SUCH A WAY AS TO MAKE THE DER   70
C     DF/DY TERM INSIGNIFICANT, IT IS POSSIBLE THAT IT MAY FAIL FOR THATDER   80
C     REASON.  IT MAY FAIL FOR JTHER REASONS, AS WELL.  IF IT DOES FAIL DER   90
C     THE USER CAN SUPPLY HIS OWN VERSION OF DERVAL, OR MODIFY THIS     DER  100
C     ROUTINE IN SUITABLE FASHION.  THIS RJUTINE ASSUMES THAT VALUES OF DER  110
C     THE LINEAR VARIABLES HAVE BEEN SUPPLIE) PREVIOUSLY.  IF THOSE     DER  120
C     MUST BE SOLVED FOR SIMULTANEOUSLY WITH THE DERIVATIVES, THE USER  DER  130
C     MUST SUPPLY HIS OWN VERSION OF DERVAL.                            DER  140
C                                                                       DER  150
C     THE CALLING SEQUENCE FOR THIS SUBROUTINE IS                       DER  160
C                                                                       DER  170
C     CALL DERVAL(Y,YL,T,N,NY,W,KERET)                                  DER  180
C                                                                       DER  190
C     WHERE THE PARAMETERS ARE DEFINED AS FOLLOWS                       DER  200
```

```
C
C        Y         - SAME AS IN LDASUB AND SDESOL.  Y(1,I) CONTAINS THE
C                    INITIAL VALUES OF THE DEPENDENT VARIABLES.   THE
C                    VALUES JF THE DERIVATIVES ARE RETURNED IN Y(2,I).
C        YL        - SAME AS IN LDASUB AND SDESOL.  THE INITIAL VALUES OF
C                    THE LINEAR VARIABLES MUST BE SUPPLIED TO THIS VERSION
C        T         - INITIAL TIME
C        N         - SAME AS IN LDASUB, TOTAL NUMBER OF VARIABLES
C        NY        - SAME AS IN LDASUB, NUMBER OF DIFFERENTIAL EQUATIONS
C                    AND NONLINEAR VARIABLES
C        W         - SCRATCH ARRAY W FROM THE CALLING SEQUENCE OF SDESOL.
C                    THIS CAN BE USED AS NEEDED IN THIS SUBROUTINE.
C        KERET     - RETURN INDCATOR
C                          =0   NORMAL RETURN
C                          =1   ERROR RETURN
C
C     ----------------------------------------------------------------------
      DIMENSION Y(7,1), YL(1), W(1)
C
      DO 100 I=1,NY
      W(2*N+I) = AMAX1(ABS(Y(1,I)),1.)
  100 Y(3,I) = 0.
C
      HINV = 16.**20
      KERET = 0
      EPS2 = NY/1.E8
      EPS = SQRT(EPS2)
C
C
      DO 140 IT=1,10
      DO 110 I=1,NY
  110 Y(2,I) = Y(3,I)/HINV
C
      CALL DIFFUN (Y,YL,T,HINV,W)
      CALL JACMAT (Y,YL,T,HINV,-1.,NY,NY,EPS,W,W(N+1),W(3*N+1))
      NEWPW = 1
C
      DO 120 I=1,NY
  120 W(I) = W(I)*HINV
C
      CALL NUITSL (W(3*N+1),W,W(N+1),NY,NY,EPS,W(2*N+1),NEWPW,KRET)
      IF (KRET.NE.0) GO TO 170
      ER = 0.
C
      DO 130 I=1,NY
      Y(3,I) = Y(3,I)-W(N+I)
  130 ER = ER+(W(N+I)/AMAX1(ABS(Y(3,I)),1.))**2
C
```

```
      IF (ER.LT.EPS2) GO TO 150                                         DER  690
  140 CONTINUE                                                          DER  700
C                                                                       DER  710
      GO TO 170                                                         DER  720
C                                                                       DER  730
  150 DO 160 I=1,NY                                                     DER  740
  160 Y(2,I) = Y(3,I)                                                   DER  750
C                                                                       DER  760
      RETURN                                                            DER  770
  170 KERET = 1                                                         DER  780
      RETURN                                                            DER  790
      END                                                               DER  800
C
C-----------------------------------------------------------------------
C     THIS SUBROUTINE IS REQUIRED BY THE TIME INTEGRATION PACKAGE AND
C     MUST BE SUPPLIED BY THE JSER. ITS PURPJSE IS TO EVALUATE THE
C     FUNCTION AT CURRENT VALUES OF THE VARIABLES.
C-----------------------------------------------------------------------
C
C
C
      SUBROUTINE DIFFUN(Y,YL,T,HINV,DY)                                 D-005
      REAL*8 C1,C2,C4,C5
      REAL*8 BIGG,BIGGG,BIGH                                            D-010
      INTEGER*2 NAME,JA,JB,NNZ,JC,NUMNP,NUMEL,NELDOF,CONN               D-015
      COMMON/GTRY2/NAME(4615),JA(183),JB(184),NNZ,JC,NUMNP,NUMEL,NELDOF, D-020
     1CONN(128,15)                                                      D-025
      DIMENSION Y(7,1),YL(1),DY(1)                                      D-035
      COMMON/DELAY/SUM(183),PSI(93)
      COMMON/COEF/C1(183),C2(183),C4(183),C5(183)
      COMMON/SYSMT1/BIGG(4615)                                          D-050
      COMMON/SYSMT2/BIGGG(4615)                                         D-055
      COMMON/SYSMT3/BIGH(4615)
      DATA TOLD/-1.0E-30/                                               D-065
      IF(T.EQ.TOLD) GO TO 97                                            D-075
      CALL SUMT(Y,TOLD,T-TOLD)                                          D-080
   96 TOLD=T                                                            D-085
   97 CONTINUE                                                          D-090
      CALL FEEDBK(Y,TOLD)                                               D-100
      DO 110 I=1,NNZ                                                    D-110
      IF(I.GT.93) GJ TO 401
      PSI(I)=Y(1,I)                                                     D-115
  401 CONTINUE
      DY(I)=0.0                                                         D-120
      JBB=JB(I)                                                         D-125
      JE=JB(I)+JA(I)-1                                                  D-130
      DO 120 J=JBB,JE                                                   D-135
      LL=NAME(J)                                                        D-140
```

```
      IF(LL.GT.NNZ) GO TO 120                                            D-145
      DY(I)=DY(I)+BIGG(J)*(HINV*Y(2,LL)+C2(LL)*Y(1,LL)+C5(LL)*SUM(LL))+C
     11(LL)*BIGGG(J)*Y(1,LL)+C4(LL)*BIGH(J)*Y(1,LL)
  120 CONTINUE                                                           D-180
  110 CONTINUE                                                           D-185
      RETURN                                                             D-195
      END                                                                D-200
C
C-----------------------------------------------------------------------
C     THIS SUBROUTINE IS REQUIRED BY THE TIME INTEGRATION PACKAGE AND
C     MUST BE SUPPLIED BY THE USER. ITS PURPOSE IS TO EVALUATE THE J
C     MATRIX NEEDED WHEN THE CORRECTOR EQUATION IS BEING SOLVED.
C-----------------------------------------------------------------------
C
C
      SUBROUTINE JACMAT(Y,YL,T,HINV,A2,N,NY,EPS,DY,F1,PW)                J-005
      REAL*8 C1,C2,C4,C5
      REAL*8 BIGG,BIGGG,BIGH                                             J-010
      INTEGER*2 NAME,JA,JB,NNZ,JC,NUMNP,NUMEL,NELDOF,CONN                J-015
      COMMON/GTRY2/NAME(4615),JA(183),JB(184),NNZ,JC,NUMNP,NUMEL,NELDOF, J-020
     1CONN(128,15)                                                       J-025
      DIMENSION Y(7,1),YL(1),F1(1),DY(1),PW(1)                           J-035
      COMMON/COEF/C1(183),C2(183),C4(183),C5(183)
      COMMON/SYSMT1/BIGG(4615)                                           J-045
      COMMON/SYSMT2/BIGGG(4615)                                          J-050
      COMMON/SYSMT3/BIGH(4615)
      AH =-A2*HINV                                                       J-065
      DO 300 I=1,NNZ                                                     J-075
      JBB=JB(I)                                                          J-080
      JE=JB(I)+JA(I)-1                                                   J-085
      DO 310 J=JBB,JE                                                    J-090
      LL=NAME(J)
      PW(J)=BIGG(J)*(AH+C2(LL))+C1(LL)*BIGGG(J)+C4(LL)*BIGH(J)
  310 CONTINUE                                                           J-115
  300 CONTINUE                                                           J-120
      RETURN                                                             J-130
      END                                                                J-135
C
C-----------------------------------------------------------------------
C     THIS SUBROUTINE IS REQUIRED BY THE TIME INTEGRATION PACKAGE AND
C     MUST BE SUPPLIED BY THE USER. ITS PURPOSE IS TO SOLVE A LINEAR
C     SYSTEM OF EQUATIONS FOR THE NEWTON ITERATES WHEN THE CORRECTOR
C     EQUATION IS BEING SOLVED.
C-----------------------------------------------------------------------
C
C
      SUBROUTINE NUITSL(PW,DY,F1,N,NY,EPS,YMAX,NEWPW,KRET)               N-005
      INTEGER*2 K,JA,JB,NNZ,JC,NUMNP,NUMEL,NELDOF,CONN                   N-010
```

```
      COMMON/GTRY2/K(4615),JA(183),JB(184),NNZ,JC,NUMNP,NUMEL,NELDOF,CON   N-015
     1N(128,15)                                                            N-020
      DIMENSION PW(1),DY(1),F1(1),YMAX(1)                                  N-025
      DATA  SPD,SPDM1/1.05,.05/                                            N-030
      KRET = 0                                                             N-040
      EPSS = EPS**2                                                        N-045
      EPSA2 = EPSS*.0001                                                   N-050
      NOIT = N                                                             N-055
  280 DO 281 I=1,NY                                                        N-060
      LL=JB(I)                                                             N-065
  281 F1(I)=DY(I)/PW(LL)                                                   N-070
      DO 287 IT=1,NOIT                                                     N-075
      RCH = 0.                                                             N-080
      CH = 0.                                                              N-085
      DO 285 I=1,NY                                                        N-090
      LL=JB(I)                                                             N-095
      JBB=JB(I)+1                                                          N-100
      JE=JB(I)+JA(I)-1                                                     N-105
      FN = DY(I)                                                           N-110
      DO 284 J=JBB,JE                                                      N-115
      IF(K(J).LE.0.OR.K(J).GT.NY) GO TO 284                                N-120
      FN=FN-PW(J)*F1(K(J))                                                 N-125
  284 CONTINUE                                                             N-130
      FN=FN/PW(LL)                                                         N-135
      FN = FN*SPD - SPDM1*F1(I)                                            N-140
      ACH = F1(I) - FN                                                     N-145
      CH = CH + (ACH/YMAX(I))**2                                           N-150
  282 RCH = RCH + (ACH/AMAX1(ABS(FN),EPS))**2                              N-155
  285 F1(I) = FN                                                           N-160
      IF(RCH.LT.EPSS)GO TO 288                                             N-165
      IF(CH.LE.EPSA2)GO TO 288                                             N-170
  287 CONTINUE                                                             N-175
      KRET = 3                                                             N-180
  288 CONTINUE                                                             N-185
      RETURN                                                               N-190
      END                                                                  N-195
C
C----------------------------------------------------------------------
C     THIS SUBROUTINE CALCJLATES THE GGG ELEMENT MATRIX USING 21 POINTS
C     OF INTEGRATION. IT THEN PUTS THE GGG ELEMENT MATRIX INTO THE BIGH
C     SYSTEM MATRIX.
C----------------------------------------------------------------------
C
C
      SUBROUTINE FEEDBK(Y,TOLD)                                            T-005
      REAL*8 GGG,BIGH                                                      T-010
      REAL*8 WL,WS,S,CL1,CL2,CL3,FN,DL1,DL2,DL3,DS,DETJ                    T-015
      INTEGER*2 NAME,JA,JB,NNZ,JC,NUMNP,NUMEL,NELDOF,CONN                  T-020
```

```
      COMMON/GTRY2/NAME(4615),JA(183),JB(184),NNZ,JC,NUMNP,NUMEL,NELDOF,   T-025
     1CONN(128,15)                                                         T-030
      COMMON/GTRY1/X(505),P(505),Z(505)                                    T-035
      COMMON/XOCAL/XX(15),YY(15),ZZ(15),PPSI(15)                           T-045
      DIMENSION Y(7,1)                                                     T-040
      COMMON/TEMP/C3(183),C6(15)
      COMMON/GGGMAT/GGG(15,15)                                             T-055
      COMMON/SYSMT3/BIGH(4615)
      COMMON/GAUSS/WL(7),WS(5),S(5),CL1(7),CL2(7),CL3(7),FN(15,21),DL1(1   T-065
     15,21),DL2(15,21),DL3(15,21),DS(15,21),DETJ(21)                       T-070
      DO 300 I=1,JC
      BIGH(I)=0.0                                                          T-085
  300 CONTINUE                                                             T-090
      NNAAA=7                                                              T-095
      NNSSS=3                                                              T-100
      DO 800 L=1,NUMEL
      DO 600 I=1,15                                                        T-120
      DO 610 J=1,15                                                        T-125
      GGG(I,J)=0.0                                                         T-130
  610 CONTINUE                                                             T-135
  600 CONTINUE                                                             T-140
      DO 805 JJ=1,15                                                       T-145
      NN=CONN(L,JJ)                                                        T-155
      XX(JJ)=X(NN)                                                         T-160
      YY(JJ)=P(NN)                                                         T-165
      ZZ(JJ)=Z(NN)                                                         T-170
      IF(NN.GT.NNZ) GO TO 803                                              T-175
      C6(JJ)=C3(NN)
      PPSI(JJ)=Y(1,NN)                                                     T-180
      GO TO 805                                                            T-185
  803 PPSI(JJ)=0.0                                                         T-190
      C6(JJ)=0.0
  805 CONTINUE                                                             T-195
      A5=2.0/(ZZ(1)-ZZ(15))                                                T-200
      DETJJ=0.0                                                            T-205
      DO 802 K=1,NNSSS                                                     T-210
      DETJ1=0.0                                                            T-215
      DO 798 M=1,NNAAA                                                     T-220
      IF(K.EQ.1) GO TO 111                                                 T-225
      IF(K.EQ.2) GO TO 222                                                 T-230
      IF(K.EQ.3) GO TO 333                                                 T-235
      GO TO 788                                                            T-240
  111 N=M                                                                  T-245
      GO TO 788                                                            T-250
  222 N=M+7                                                                T-255
      GO TO 788                                                            T-260
  333 N=M+14                                                               T-265
  788 CONTINUE                                                             T-270
```

```
      T1=DL1(1,N)*XX(1)+DL1(2,N)*XX(2)-DL3(4,N)*XX(4)-DL3(5,N)*XX(5)      T-275
      T2=DL1(7,N)*XX(7)-DL3(9,V)*XX(9)+DL1(10,N)*XX(10)+DL1(11,N)*XX(11)  T-280
      T3=(DL1(6,N)-DL3(6,N))*XX(6)-DL3(13,N)*XX(13)-DL3(14,N)*XX(14)      T-285
      T4=(DL1(15,N)-DL3(15,N))*XX(15)                                      T-290
      DJ11=T1+T2+T3+T4                                                     T-295
      D1=DL1(1,N)*YY(1)+DL1(2,N)*YY(2)-DL3(4,N)*YY(4)-DL3(5,N)*YY(5)      T-300
      D2=DL1(7,N)*YY(7)-DL3(9,V)*YY(9)+DL1(10,N)*YY(10)+DL1(11,N)*YY(11)  T-305
      D3=(DL1(6,N)-)L3(6,N))*YY(6)-DL3(13,N)*YY(13)-DL3(14,N)*YY(14)      T-310
      D4=(DL1(15,N)-)L3(15,N))*YY(15)                                      T-315
      DJ12=D1+D2+D3+D4                                                     T-320
      T5=DL2(2,N)*XX(2)+DL2(3,V)*XX(3)-DL3(5,N)*XX(5)-DL3(6,N)*XX(6)      T-325
      T6=DL2(8,N)*XX(8)-DL3(9,N)*XX(9)+(DL2(4,N)-DL3(4,N))*XX(4)          T-330
      T7=DL2(11,N)*XX(11)+DL2(12,N)*XX(12)+(DL2(13,N)-DL3(13,N))*XX(13)   T-335
      T8=-DL3(14,N)*XX(14)-DL3(15,N)*XX(15)                                T-340
      DJ21=T5+T6+T7+T8                                                     T-345
      D5=DL2(2,N)*YY(2)+DL2(3,V)*YY(3)-DL3(5,V)*YY(5)-DL3(6,N)*YY(6)      T-350
      D6=DL2(8,N)*YY(8)-DL3(9,N)*YY(9)+(DL2(4,N)-DL3(4,N))*YY(4)          T-355
      D7=DL2(11,N)*YY(11)+DL2(12,N)*YY(12)+(DL2(13,N)-DL3(13,N))*YY(13)   T-360
      D8=-DL3(14,N)*YY(14)-DL3(15,N)*YY(15)                                T-365
      DJ22=D5+D6+D7+D8                                                     T-370
      DETJ(N)=DJ11*DJ22-DJ12*DJ21                                          T-375
      DETJ1=DETJ(N)*WL(M)+DETJ1                                            T-380
  798 CONTINUE                                                             T-385
      DETJJ=DETJJ+WS(K)*DETJ1                                              T-390
  802 CONTINUE                                                             T-395
      DO 700 J=1,15                                                        T-400
      DO 710 I=1,15                                                        T-405
      FHH=0.0                                                              T-410
      DO 806 K=1,NNSSS                                                     T-415
      FH=0.0                                                               T-420
      DO 799 M=1,NNAAA                                                     T-425
      TT1=0.0                                                              T-430
      TT2=0.0                                                              T-435
      TT3=0.0                                                              T-440
      IF(K.EQ.1) GO TO 1111                                                T-445
      IF(K.EQ.2) GO TO 2222                                                T-450
      IF(K.EQ.3) GO TO 3333                                                T-455
      GO TO 7888                                                           T-480
 1111 N=M                                                                  T-485
      GO TO 7888                                                           T-490
 2222 N=M+7                                                                T-495
      GO TO 7888                                                           T-500
 3333 N=M+14                                                               T-505
 7888 CONTINUE                                                             T-550
  770 TT1=FN(1,N)*PPSI(1)*C6(1)+FN(2,N)*PPSI(2)*C6(2)+FN(3,N)*PPSI(3)*C6
     1(3)+FN(4,N)*PPSI(4)*C6(4)+FN(5,N)*PPSI(5)*C6(5)+FN(6,N)*PPSI(6)*C6
     1(6)+FN(7,N)*PPSI(7)*C6(7)+FN(8,N)*PPSI(8)*C6(8)
      TT2=FN(9,N)*PPSI(9)*C6(9)+FN(10,N)*PPSI(10)*C6(10)+FN(11,N)*PPSI(1
```

```
     11)*C6(11)+FN(12,N)*PPSI(12)*C6(12)+FN(13,N)*PPSI(13)*C6(13)+FN(14,
     1N)*PPSI(14)*C6(14)+FN(15,N)*PPSI(15)*C6(15)
      TT3=TT1+TT2+1.0                                                   T-580
      FH=FH+WL(M)*FN(J,N)*FN(I,N)*ALOG(TT3)*DETJ(N)                     T-585
  799 CONTINUE                                                          T-590
      FHH=WS(K)*FH+FHH                                                  T-595
  806 CONTINUE                                                          T-600
      GGG(J,I)=FHH/A5                                                   T-605
  710 CONTINUE                                                          T-610
  700 CONTINUE                                                          T-615
C    INSERT INTO SYSTEM MATRIX                                          T-620
      DO 20 K=1,15                                                      T-625
      KK=CONN(L,K)                                                      T-630
      IF(KK.GT.NNZ) GO TO 20                                            T-635
      KKK=JA(KK)                                                        T-640
      LLL=JB(KK)-1                                                      T-645
      DO 10 I=1,15                                                      T-650
      II=CONN(L,I)                                                      T-655
      DO 91 M=1,KKK                                                     T-660
      MM=LLL+M                                                          T-665
      KKM=NAME(MM)                                                      T-670
      IF(II.EQ.KKM)GO TO 92                                             T-675
   91 CONTINUE                                                          T-680
      GO TO 10                                                          T-685
   92 CONTINUE                                                          T-690
      BIGH(MM)=BIGH(MM)+GGG(K,I)                                        T-695
   10 CONTINUE                                                          T-700
   20 CONTINUE                                                          T-705
  800 CONTINUE                                                          T-710
      RETURN                                                            T-715
      END                                                               T-720
C
C-----------------------------------------------------------------------
C     THE CUMULATIVE CONTRIBUTION OF THE DELAYED NEUTRON FLUX IS
C     CALCULATED BY THIS SUBROUTINE.
C-----------------------------------------------------------------------
C
      SUBROUTINE SUMT(Y,T,H)                                            S-005
      DIMENSION Y(7,1)                                                  S-010
      COMMON/DELAY/SUM(183),PSI(93)
      Q=0.4349710
      TC=-Q*H                                                           S-030
      DO 100 I=1,93                                                     S-040
      SUM(I)=SUM(I)*EXP(-Q*H)+0.50*H*(PSI(I)*EXP(TC)+Y(1,I))
  100 CONTINUE                                                          S-050
      RETURN                                                            S-055
      END                                                               S-060
```

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